Line energy of dislocations

From the elastic stress states we obtain the elastic energy $E^{\odot}$, which is contained in the screw dislocation. In order to create a dislocation (e. g. screw-) a movement of the hollow cylinder by the amount of $\vec{b}$ in z-direction is needed (see Fig. 1.22); it gives $dA = dr \cdot dz = dr \cdot db$ as $z = 0 \ldots b$ and only $\tau_{\theta z}$ appears in the stress matrix. As θ is kept konstant and as we know the integration limits of z it holds:

$$E_{el}^{\odot} = \frac{1}{2}\int_{r_0}^{R}\left(\int_0^b \tau_{\theta z} \cdot dz\right) dr = \int_{r_0}^{R}\left(\frac{Gb}{4\pi r} z\right)\Bigg|_0^b dr = \int_{r_0}^{R} \frac{Gb^2}{4\pi r} dr = \frac{Gb^2}{4\pi} \ln r \Bigg|_{r_0}^{R} = \frac{Gb^2}{4\pi} \ln \frac{R}{r_0} \simeq \frac{Gb^2}{4\pi} \ln \frac{R}{b}$$

Analogously (without proof) for edge dislocations:

$$E_{el}^{\perp} \simeq \frac{Gb^2}{4\pi(1-\nu)} \ln \frac{R}{b}$$

Discussion of the results

a.)The line energy E_{el} has the dimension of a force [N], i. e. $\frac{\text{energy}}{\text{length}}$ $\left[\frac{\text{Nm}}{\text{m}}\right]$!

b.)values for r_0, R:

$r \simeq b$ (see above)

$R =:$ outer cutting radius $\simeq$ mean spacing of dislocations $\Rightarrow R \approx \frac{1}{\sqrt{\rho}} \overbrace{\simeq}^{\rho \simeq 10^{14} m^{-2}} 0.1\mu\text{m}$

$\Rightarrow \frac{R}{r_0} \simeq \frac{10^{-7} m}{10^{-10} m} \simeq 10^3 \Rightarrow$ stress field of single dislocation is long range!

c.)estimate of energy

$E_{el}^{\odot} \simeq \frac{Gb^2}{12} \ln(10^3) \simeq \frac{1}{2} Gb^2$ ($\simeq$ 4eV $\gg$ energy needed for producing vacancies, 1eV)

$E_{el}^{\perp} \simeq \frac{Gb^2}{12(1-\nu)} \ln(10^3) = \frac{3}{4} Gb^2 = \frac{3}{2} E_{el}^{\odot}$ as $\nu \simeq \frac{1}{3}$

⇒The line energy of an edge dislocation is greater that that of a screw dislocation. Dislocation loops are ellypticaly (and not in form of a circle) with short edge- and long screw segments.

⇒Dislocations cannot produced by thermal fluctuations (4 eV ≫ 1 eV) but must be procuced mechanically

1.2.5 Interaction of dislocations

Peach-Koehler-equation

We remember the picture of conservative dislocation movement (shear of a crystal):

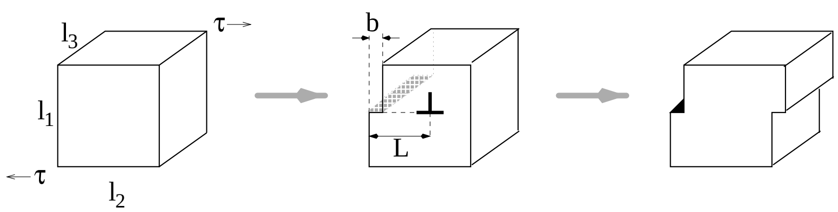


Figure 1.25: shearing of a crystal - conservative dislocation movement, see also Fig. 1.24.

Consider the supplied work A due to gliding (Force= $\tau l_1 \cdot l_3$):

$$A = \tau l_2 \cdot l_3 \cdot b \quad \text{(hatched area in Fig. 1.25)}$$

On the other hand, the force upon the dislocation per line length yields

$$A = K \cdot l_3 \cdot l_2 \quad \text{(dislocation moved by } l_2\text{)}$$

A comparison of the coefficients yields: $K = \tau \cdot b$ This is the 1-dimensional representation of the Peach-Koehler-equation. The general form for any desired stress state σ yields

$$\vec{K} = (\sigma \vec{b}) \times \vec{ds}$$

of the Peach-Koehler-equation. For the interaction of dislocations we consider some special cases:

screw dislocations

a.)two parallel screw dislocations, i. e. $\vec{b_1} = \vec{b_2}$ (and $|\vec{b_1}| = |\vec{b_2}|$!)

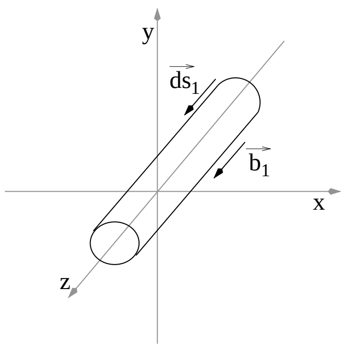
K_{12} (force, which dislocation 1 with stress field σ_1 exerts on dislocation 2 with Burger's vector $\vec{b_2}$ and line element $\vec{ds_2}$)

$$K_{12} = (\sigma_1 \cdot b_2) \times \vec{ds_2}$$

$$K_{12} = \left[\begin{pmatrix} 0 & 0 & \tau_{xz} \\ 0 & 0 & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & 0 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ b_2 \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$K_{12} = \begin{pmatrix} \tau_{xz} b_2 \\ \tau_{yz} b_2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \tau_{yz} b_2 \\ -\tau_{xz} b_2 \\ 0 \end{pmatrix}; \qquad \begin{aligned} \tau_{yz} &= +\frac{Gb}{2\pi}\frac{x}{x^2+y^2} \\ \tau_{xz} &= -\frac{Gb}{2\pi}\frac{y}{x^2+y^2} \end{aligned}$$

$$= \begin{pmatrix} \frac{Gb_1b_2}{2\pi}\frac{x}{x^2+y^2} \\ \frac{Gb_1b_2}{2\pi}\frac{y}{x^2+y^2} \\ 0 \end{pmatrix} \underbrace{=}_{\vec{b_1}=\vec{b_2}} \begin{pmatrix} \frac{Gb^2}{2\pi}\frac{x}{x^2+y^2} \\ \frac{Gb^2}{2\pi}\frac{y}{x^2+y^2} \\ 0 \end{pmatrix}$$



$\Rightarrow \left.\begin{matrix} K_x > 0 \\ K_y > 0 \end{matrix}\right\}$ always repelling forces of two parallel screw dislocations

$K_z = 0$ no force in direction of dislocation line (i. e. z-direction)

b.)two antiparallel screw dislocations, i. e. $\vec{b_1} = -\vec{b_2}$

$\Rightarrow \left.\begin{matrix} K_x = -\frac{Gb^2}{2\pi}\frac{x}{x^2+y^2} < 0 \\ K_y = -\frac{Gb^2}{2\pi}\frac{y}{x^2+y^2} < 0 \end{matrix}\right\}$ always attractive interaction of two antiparallel screw dislocations

$K_z = 0$ $\rightarrow$ Annihilation

edge dislocations

a.)two parallel edge dislocations:

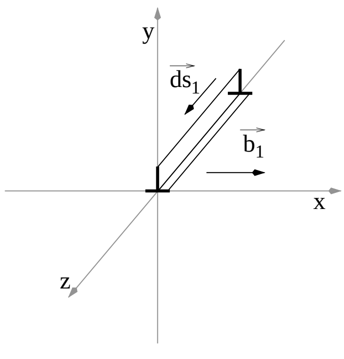
$$K_{12} = \left[\begin{pmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_{yy} & 0 \\ 0 & & \sigma_{zz} \end{pmatrix} \cdot \begin{pmatrix} b_2 \\ 0 \\ 0 \end{pmatrix}\right] \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$K_{12} = \begin{pmatrix} \sigma_{xx} b_2 \\ \tau_{xy} b_2 \\ 0 \end{pmatrix} \times \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \tau_{xy} b_2 \\ -\sigma_{xx} b_2 \\ 0 \end{pmatrix}$$

$$\Rightarrow K_x = \frac{Gb_1b_2}{2\pi(1-\nu)}\frac{x(x^2-y^2)}{(x^2+y^2)^2} \underbrace{=}_{\vec{b_1}\vec{b_2}} \frac{Gb^2}{2\pi(1-\nu)}\frac{x(x^2-y^2)}{(x^2+y^2)^2}$$

$$K_y = \frac{Gb_1b_2}{2\pi(1-\nu)}\frac{y(3x^2+y^2)}{(x^2+y^2)^2} \overbrace{=} \frac{Gb^2}{2\pi(1-\nu)}\frac{y(3x^2+y^2)}{(x^2+y^2)^2}$$

$K_z = 0$ $\Rightarrow$no force in direction of the dislocation line

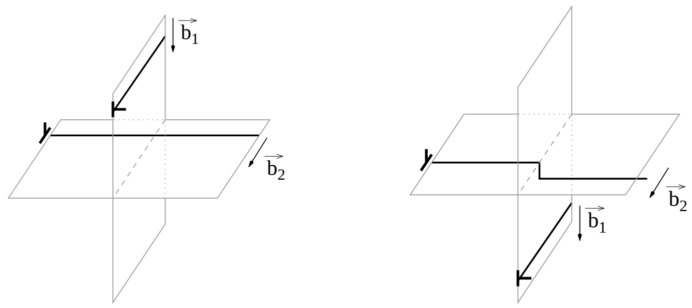


<u>Intersection:</u> *Interaction of two dislocations*

In general when dislocations interact a break in the dislocation line can be created. A dislocation moving in its slip plane will intersect other dislocations crossing the slip plane. The intersection produces a break, of Burger's vector length, in the dislocation line.
<u>Remember</u>: Although a dislocation line may have different directions it always has the same Burger's vector! A `jog` is a break in the dislocation line moving it out of the slip plane, whereas a `kink` is a break in the dislocation line which remains in the slip plane. Jog or kink form when the Burger's vector of the intersecting dislocation is normal to the dislocation line.

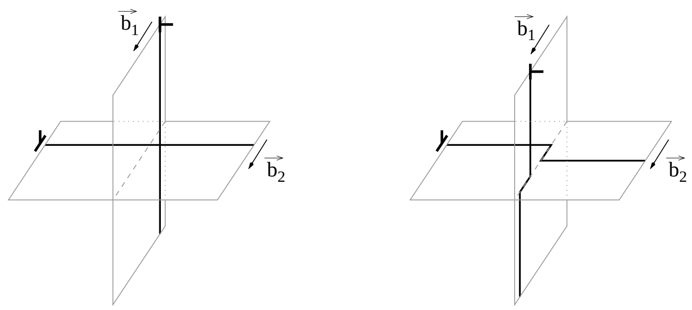
Golden rule:
A jog created in a dislocation line due to the interaction of two dislocations has the direction of the Burger's vector of the other dislocation. For e. g. the case of two edge dislocations (first case) the jog created in line $\vec{ds}_2$ has the direction of the Burger's vector $\vec{b_1}$.

Two edge dislocations with Burger's vectors normal to each other



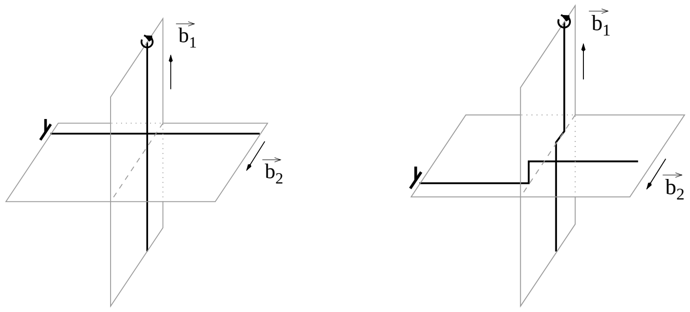
⇒ Dislocation 2 has a jog with edge configuration as $\vec{b_1} \perp \vec{ds}_2$
Nothing happens to $\vec{ds}_1$ as it is parallel to $\vec{b_2}$

Two orthogonal edge dislocations with parallel Burger's vectors



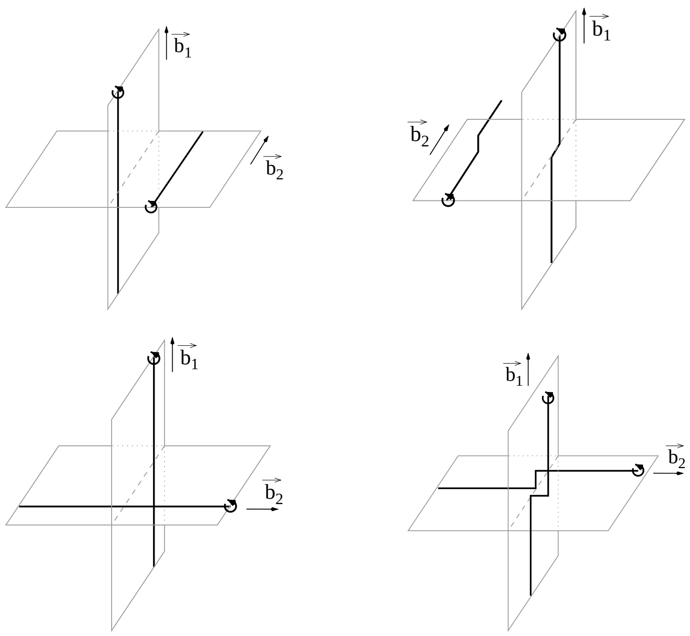
⇒ Both dislocations have kinks with screw orientation, as $\vec{b_1}$ ∥ kink_1 as well as $\vec{b_2}$ ∥ kink_2. Kinks are unstable and during glide can line-up and annihilate themselfes.

One edge and one screw dislocation



⇒ The jog (in the case of the edge dislocation) and the kink (in the case of the screw dislocation) have edge orientation as $\vec{b_i} \perp \vec{ds}_i$

Two screw dislocations



In the case of one dislocation lying parallel to the other slip plane: one dislocation has a kink with edge orientation, the other dislocation has a jog with edge orientation. When the dislocations lie orthogonal to the other slip plane each have a jog with edge orientation.
⇒ The jog and the kink have edge configuration as $\vec{b_i} \perp \vec{ds}_i$

- The only way the jog (edge orientation) can move by slip (conservative motion) is along the axis of the screw dislocation.
- For the moving screw dislocation the jog has to be dragged along (non- conservative motion → climb)
- Dislocation climb is thermally activated, so the motion of jogged screw dislocation will be temperature dependent.