

Phenomenological aspects of the metal-insulator transition in disordered systems: So, was Mott right after all?



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1. Introduction: Motivation
2. How it began: Scaling in the hopping region of amorphous $\text{Si}_{1-x}\text{Cr}_x$, $\sigma(T,x) = \sigma(T/T_0(x))$
3. Generalisation: Comparative study of a- $\text{Si}_{1-x}\text{Ni}_x$
4. Possible causes of the MIT in three-dimensional systems
5. Excursion to a two-dimensional system: Indications for a finite-temperature transition
6. Conclusions & challenges

University of Minnesota, Minneapolis, 04/08/2010

1. Introduction: Motivation

Basic definition relates to direct-current zero-temperature conductivity, $\sigma (T \rightarrow 0)$:

insulator: $\sigma (T \rightarrow 0) = 0$ versus metal: $\sigma (T \rightarrow 0) = \text{finite}$

Consequence: extrapolation needed. Standard approach:

$$\sigma (T, x) = a(x) + b(x) \cdot T^{1/2} \quad (\text{Altshuler and Aronov, 1979})$$

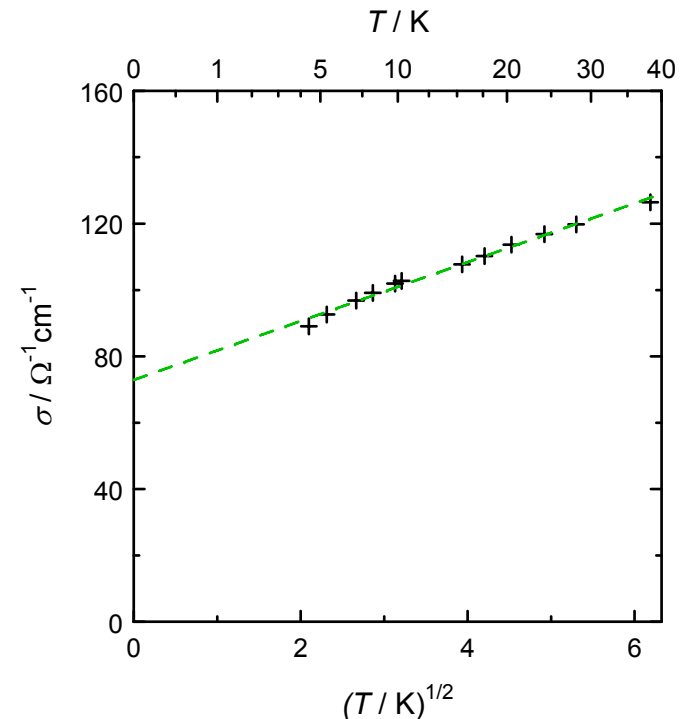
Example: amorphous $\text{Si}_{0.87}\text{Cr}_{0.18}$

Range 4 – 40 K: Interpolation works fine,
so extrapolation seems to be reasonable,

$$\sigma (T \rightarrow 0) = 73 \, \Omega^{-1} \text{cm}^{-1}$$

sample is obviously metallic

Data from J. Phys. C 18 (1985) 3337

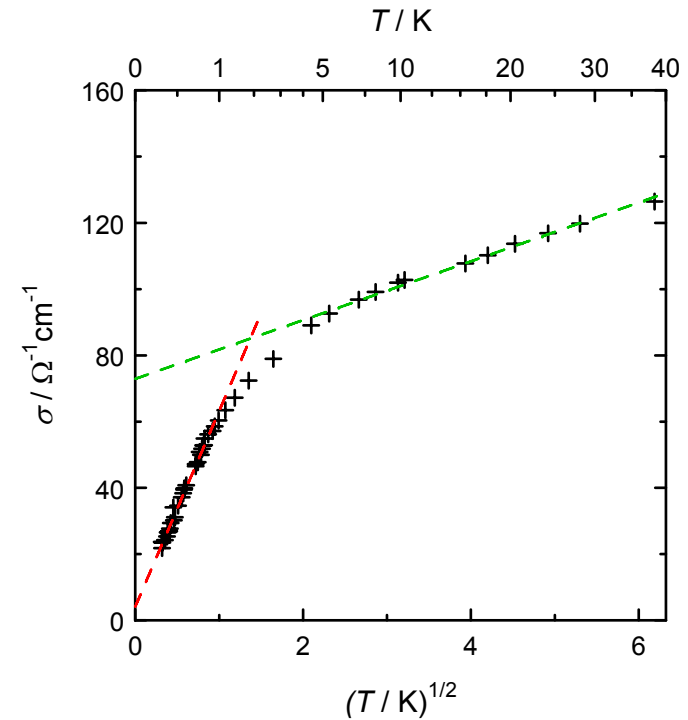


Extrapolation check by means of
dilution refrigerator:

Range 100 mK — 1 K: Again, interpolation
seems to work fine. Extrapolation yields

$$\sigma(T \rightarrow 0) = 4.1 \Omega^{-1} \text{cm}^{-1},$$

by factor 18 smaller than previous value.
Sample seems to be positioned just at
metal-insulator transition (MIT).



Take home:

At limits of metallic conduction, extrapolation $T \rightarrow 0$ is often misleading.

⇒ Reason for this strange behaviour?

⇒ What do we know for sure, and what do we only believe?

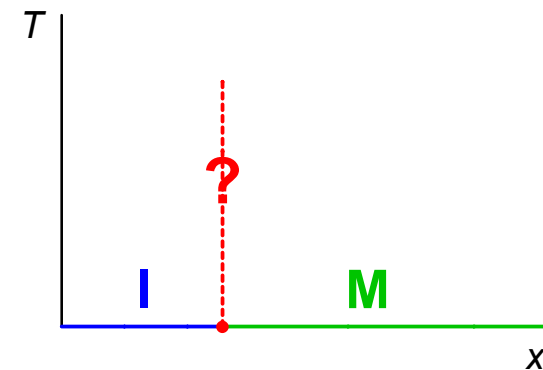
When adding to an amorphous semiconductor a transition metal, what can happen?

- A) Is there an **abrupt change** between insulating and metallic behaviour, **or** is there a **“slow” onset** of conduction? Compare structural glass transition.

Common opinion supported by studies of Anderson localization: abrupt change

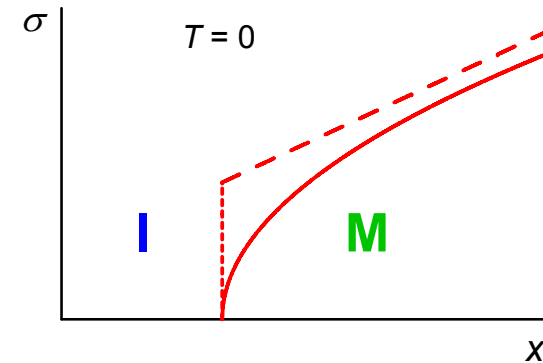
- B) Does the zero temperature phenomenon MIT have a **continuation at finite temperatures?**

Presumable majority opinion: no



- C) Does $\sigma (T \rightarrow 0, x)$ vanish continuously or discontinuously when approaching the MIT from the metallic side?

Presumable majority opinion: continuously

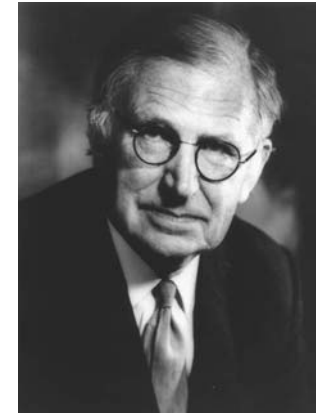


Which answers offers microscopic theory?

Mott (1972):

minimum metallic conductivity, $\sigma_{\min \text{ met}} = C \frac{e^2}{\hbar a}$ with

$$C = 0.025 \dots 0.05$$



Abrahams et al. (1979):

scaling theory \Rightarrow transition is continuous,
no transition in 2D systems.

Finkelstein (1983,84):

electron-electron interaction taken into account \Rightarrow transition is continuous

Back to experiment: Current 'majority opinion'

For three-dimensional systems, **continuous character clear**, i.e.,

$$\sigma(T=0, n) \propto (n - n_c)^\nu .$$

Exponent ν unresolved, experimental values scatter between 0.5 and 1.3.

Problem: Determination of the exponent ν needs knowledge of $\sigma(T=0, n)$, which can be determined only by means of **extrapolation**. Often used:

Altshuler and Aronov (1979): perturbation theory considering interplay of Coulomb interaction and disorder:

$$\sigma(T, n) = a(n) + b(n) \cdot T^{1/2}$$

Newson and Pepper (1986) for magnetic-field induced MIT:

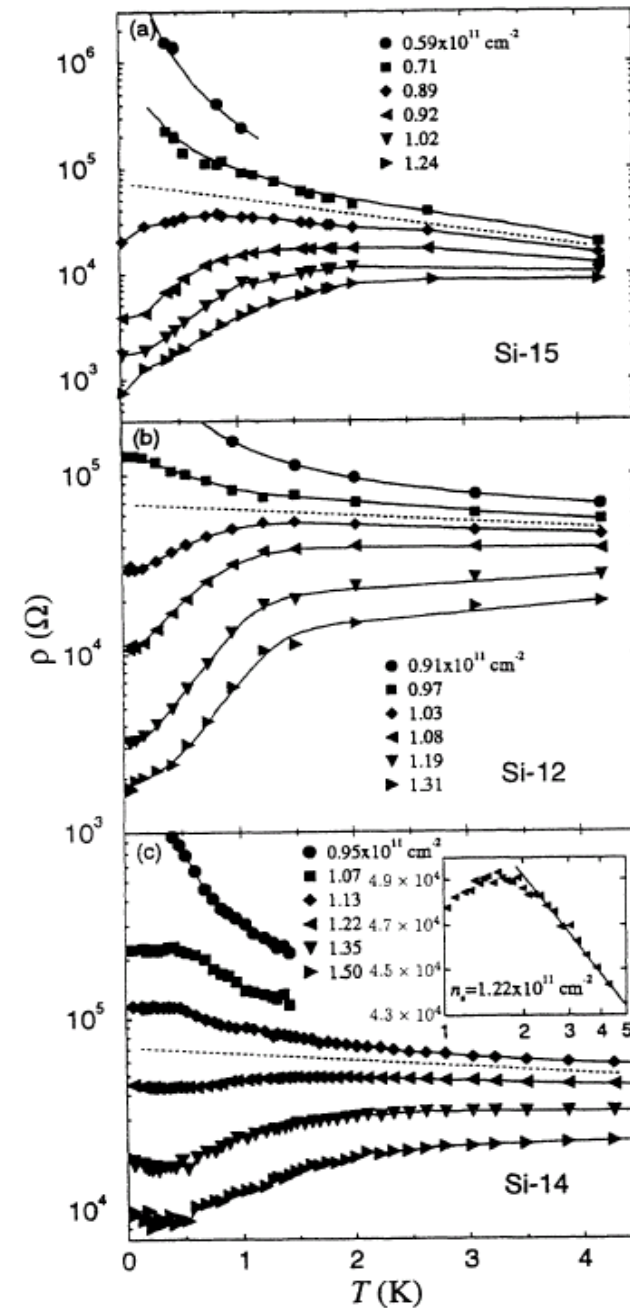
$$\sigma(T, n) = a(n) + b(n) \cdot T^{1/3}$$

In both cases: **Finite "slope" at MIT**.

However, excursion to 2D world confronts with experimental disprove of scaling theory:

Possible metal-insulator transition at $B=0$ in two dimensions,

S.V. Kravchenko et al., Phys. Rev. B **50**, 8039 (1994)



Note difference to 3D:

Vanishing of slope is used as criterion for MIT

Back to 3D: What would happen if slope = 0 at MIT?

Citation from classical paper:

“The evidence is thus very strong that at zero temperature there is a true discontinuity in ρ as the compensation increases. **But this is not a first order transition in other properties; the activation energy for hopping shows no discontinuity.**”

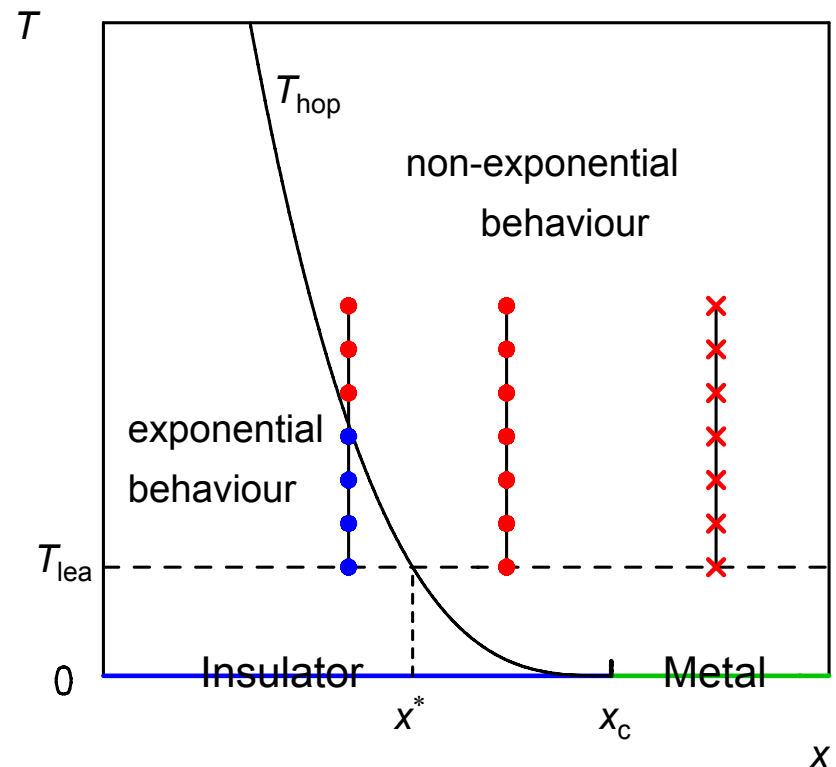
N.F. Mott, Phil. Mag. 26 (1972) 1015; in part. 1021f.

⇒ **Exponential behaviour can only be expected as long as $T \ll T_{\text{hop}}$.**

Above T_{hop} : comparably flat $\sigma(T)$.

This non-exponential hopping and metallic transport can easily be mixed up.

In concentration region $x^* < x < x_c$, only non-exponential $\sigma(T)$ observed.



Take home:

It is impossible to study MIT at low temperatures in case characteristic hopping energy vanishes as MIT is approached.

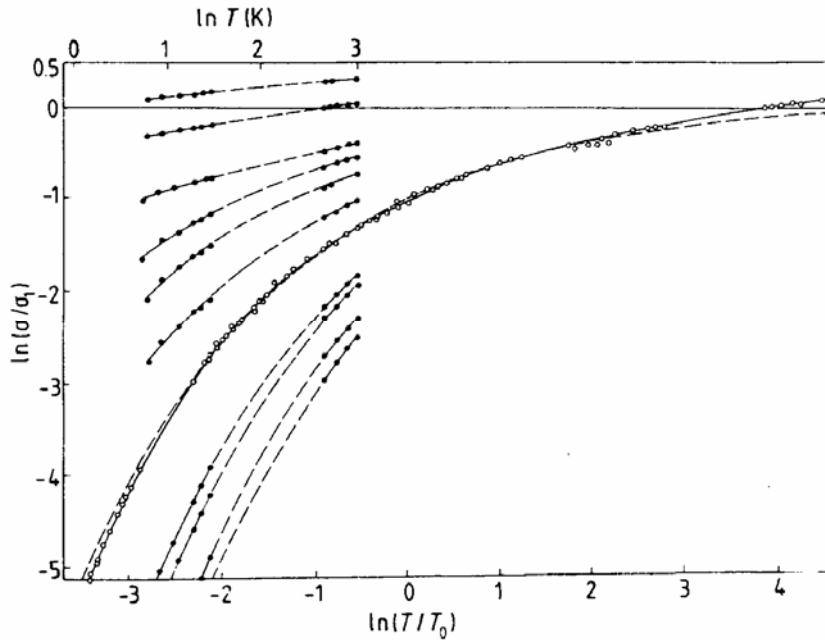
⇒ Since broadly accepted theory of non-exponential hopping seems to be missing, **unbiased approaches are highly desirable** to discriminate between metallic and activated conduction.

Possibilities:

consideration of derivatives and / or

search for universal relations between various characteristics

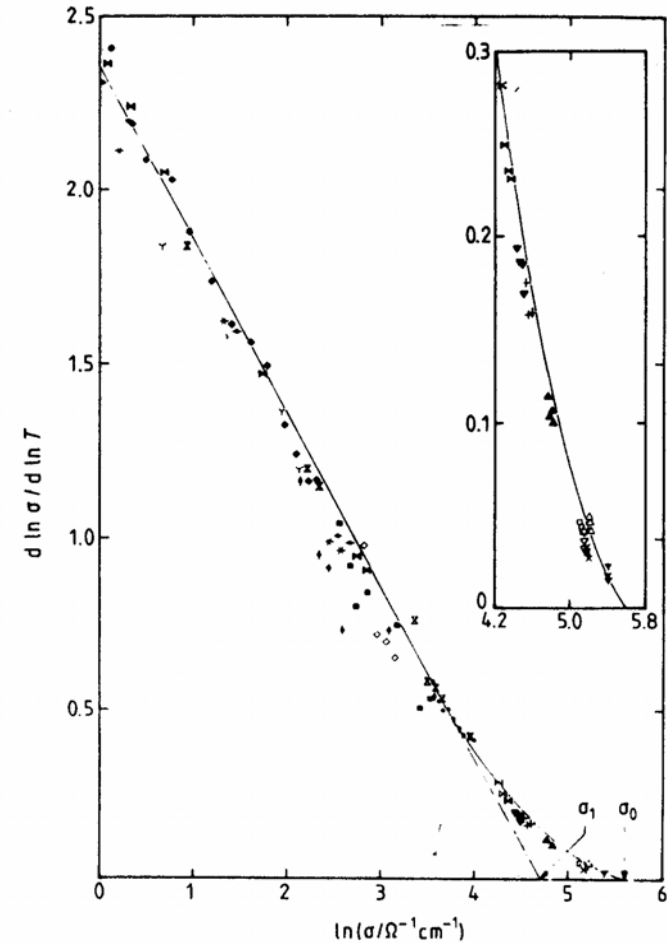
2. How it began: Scaling in the hopping region of amorphous $\text{Si}_{1-x}\text{Cr}_x$, $\sigma(T,x) = \sigma(T/T_0(x))$



No metallic conduction in scaling region of σ !

AM, D. Elefant, A. Heinrich, R. Müller, J. Schumann,
H Vinzelberg, G Zies, J. Phys. C **16** (1983) 6491

AM, H Vinzelberg, C. Gladun, A Heinrich, D Elefant,
J Schumann, G Zies, J. Phys. C **18** (1985) 3337



$d \ln \sigma / d \ln T$ is a function of σ only

What about other materials?

Neutron-transmutation doped Ge:Ga:

Efros-Shklovskii hopping where

characteristic temperature vanishes and **prefactor is constant** as MIT is approached,

but conclusion on discontinuity at $T = 0$ not drawn.

A.G. Zabrodskii and K.N. Zinov'eva,
Soviet Phys. JETP 59, 425 (1984):

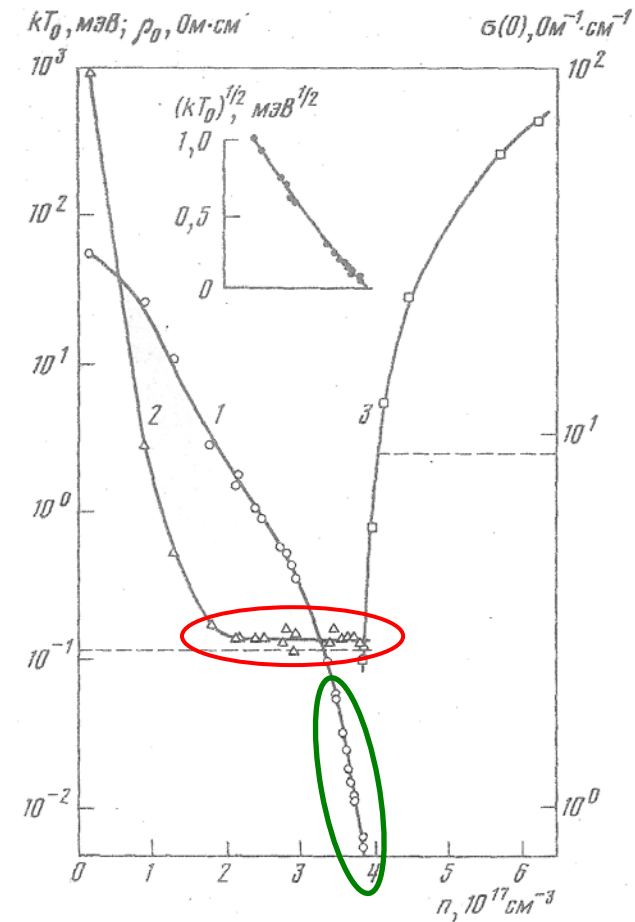
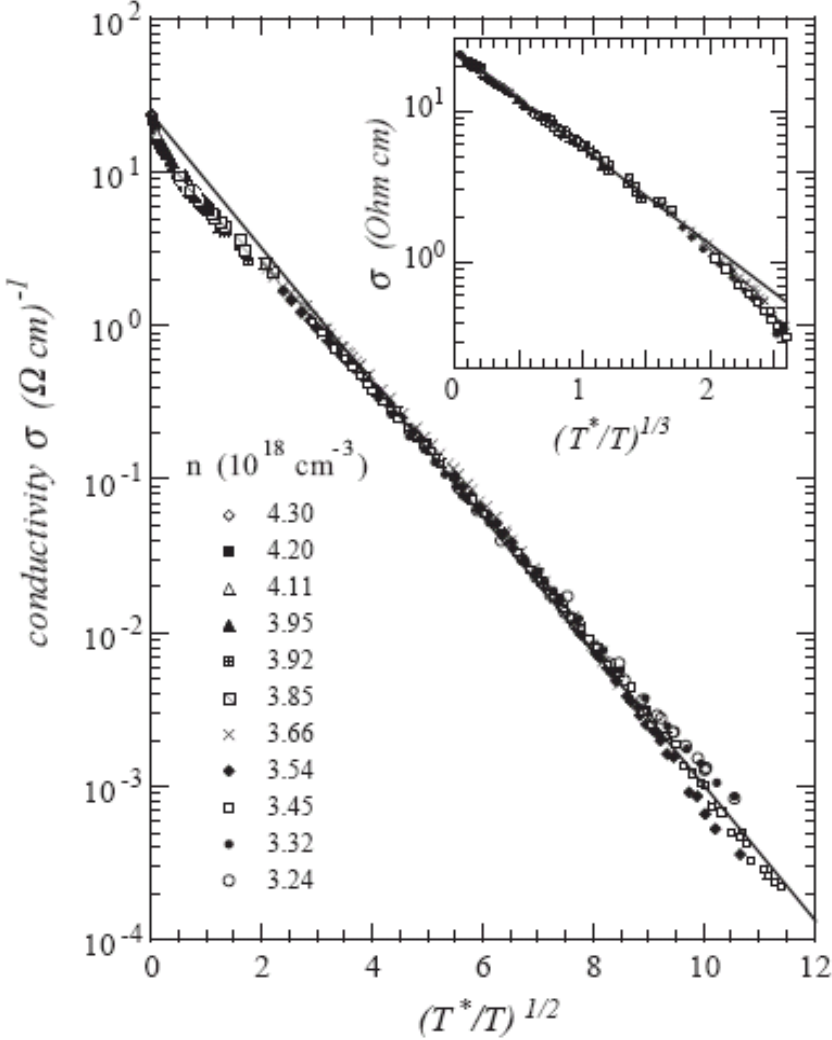
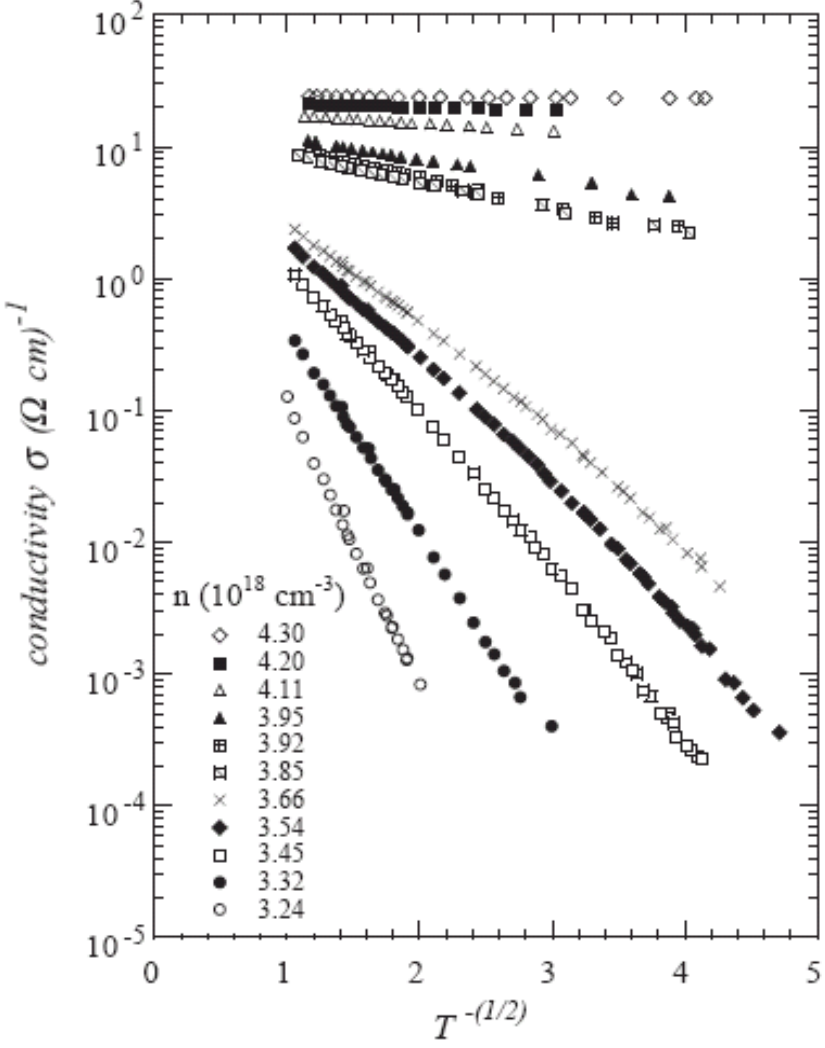


Рис. 3. Критическое поведение проводимости: 1 — T_0 , 2 — ρ_0 , 3 — $\sigma(0)$. Пунктир: σ_M^{-1} (слева) и σ_M (справа)

Crystalline Si:B by M.P. Sarachik and P. Dai, Europhys. Lett. 59, 100 (2002):



Scaling analysis of T dependence of σ in the hopping region = powerful tool

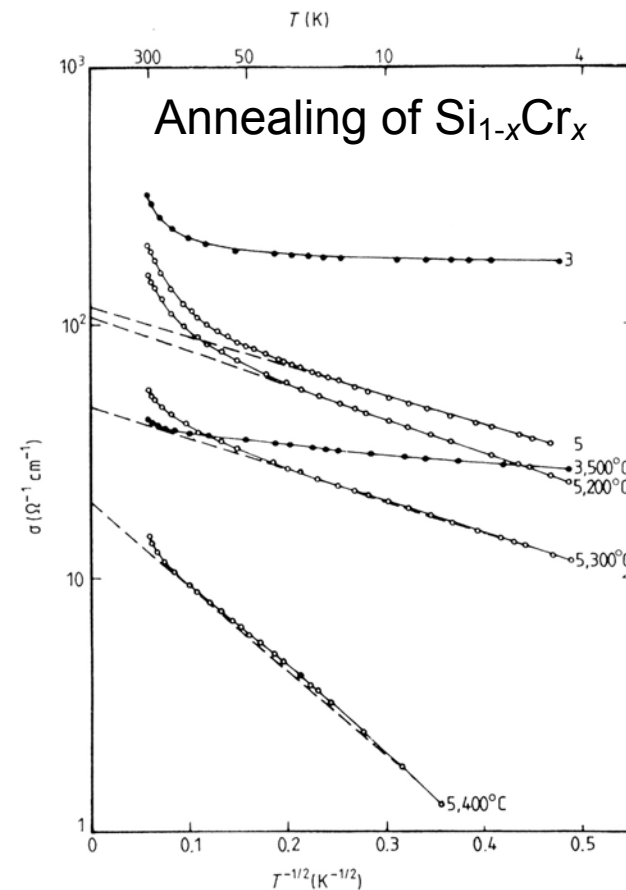
Scaling + monotonous concentration dependence

⇒ existence of finite minimum metallic conductivity

However, scaling analysis fails if
material is not homogenous,
i.e., for granular systems,

or if

second conduction mechanism
is relevant.



3. Generalisation: Comparative study of amorphous $\text{Si}_{1-x}\text{Ni}_x$

- Aim: Experimental study of metal-insulator transition **without fits**,
data analysis focussing on $w(T,x) = \partial \ln \sigma(T,x) / \partial \ln T$ instead
- Collaboration: C. Frenzel, R. Thielsch, H.-D. Bauer, V. Hoffmann, T. Krieg,
N. Matz, H. Vinzelberg, IFW Dresden,
R. Rosenbaum, Tel Aviv University;
R. Grötzschel, Research Center Rossendorf,
M. Witcomb, University of the Witwatersrand,
C.J. Adkins, Cavendish Laboratory, Cambridge,
M. Schreiber, Technical University Chemnitz
- Sponsor: German-Israeli Foundation for Scientific Research and Development
- For details: Physical Review B **60**, 14209 (1999)

Why do we consider the logarithmic derivative $w(T) = d \ln \sigma / d \ln T$?

A) Metallic sample

$$\sigma = a + b T^p \text{ with } a > 0 \Rightarrow w = p b T^p / (a + b T^p) \rightarrow 0$$

B) Limiting case in case of continuous transition

$$\sigma = b T^p \Rightarrow w = p$$

C) Activated conduction

$$\sigma = A \exp\{-(T_0/T)^q\} \Rightarrow w = q (T_0/T)^q \rightarrow \infty$$

Hence, concerning broad classes of T dependences, qualitative classification possible.

Remark: Principal extrapolation problem cannot be solved.

Sample Preparation

Two variants of electron-beam evaporation:

1. Evaporation of alloy from one crucible (Dresden)

- Advantages:
- high lateral homogeneity,
 - composition can be varied by changing power

2. Gradient deposition by evaporation of elements from two crucibles (Tel Aviv)

- Advantages:
- composition can be related to geometry,
 - same preparation conditions for whole sample set

Characterization

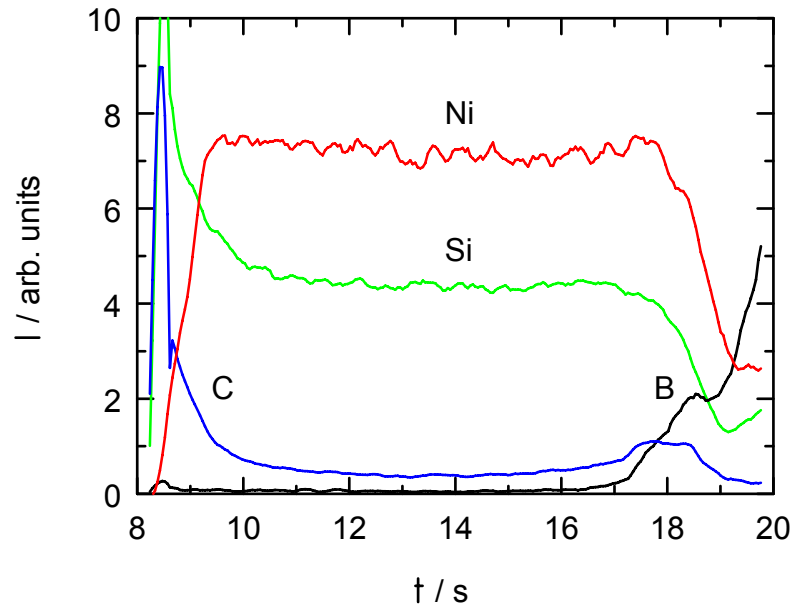
Electron microscopy + EDX,

RBS,

GDOES (Glow-Discharge-Optical-Emission Spectroscopy) for homogeneity check,

STM

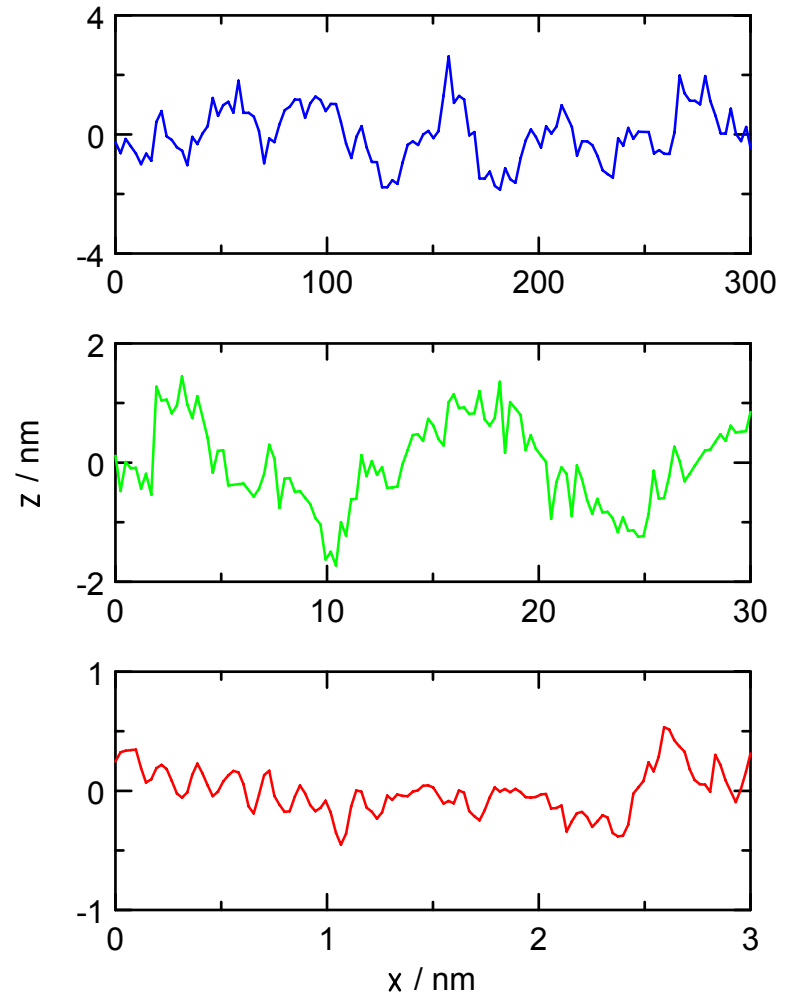
GDOES:



Ni line confirms vertical homogeneity.

Variation of Si line at ignition arises from surface contamination.

STM:



Conductivity measurements

Temperature ranges

- (a) 2 — 300 K: Self-built cryostat insert (bell technique),
stability: $\Delta T/T < 5 \cdot 10^{-4}$.
- (b) 0.45 — 4.2 K: RMC ^3He - cryostat
- (c) 35 — 900 mK: Kelvinox 300 from Oxford Instruments

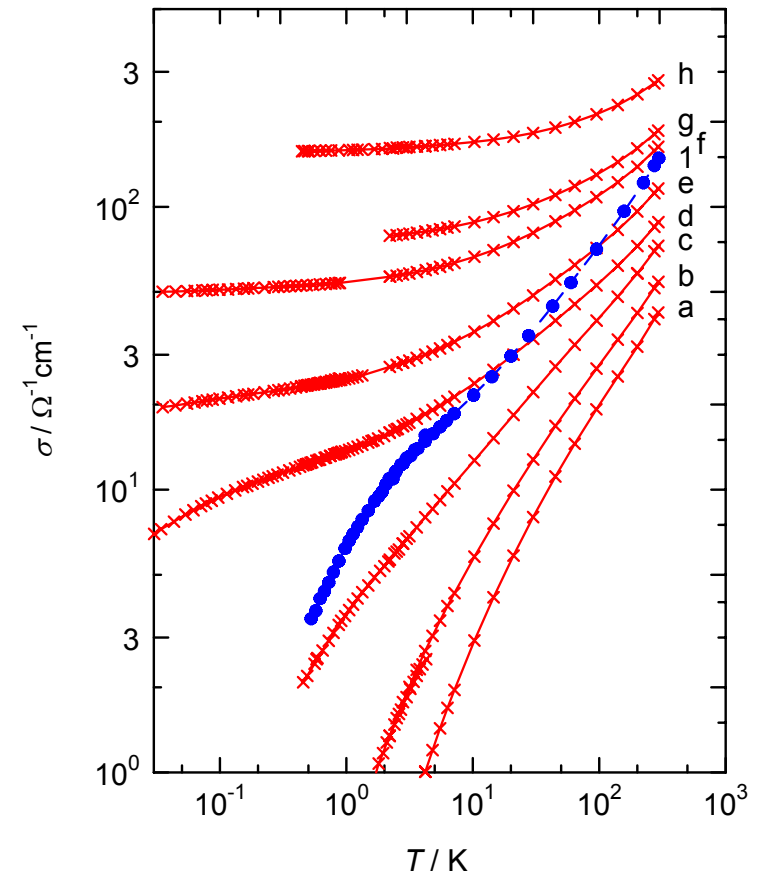
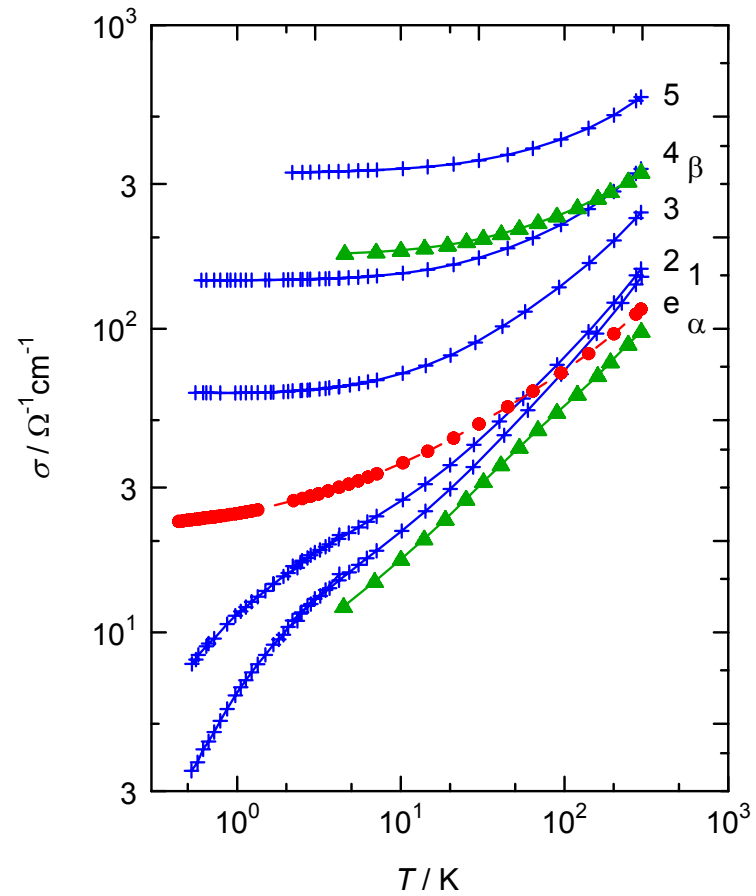
Data evaluation

Numerical procedure to obtain values of $w(T,x) = \partial \ln \sigma(T,x) / \partial \ln T$:

Piecewise linear approximation of $\ln \sigma$ as function of $\ln T$ where **appropriate reference point** ensures high accuracy
(= generalisation of midpoint rule).

Idea behind: choose reference point so that parabola is exactly differentiated.

Overview



-- = Dresden (evaporated), -- = Tel Aviv (evap.), -- = Isobe et al. (sputtered)

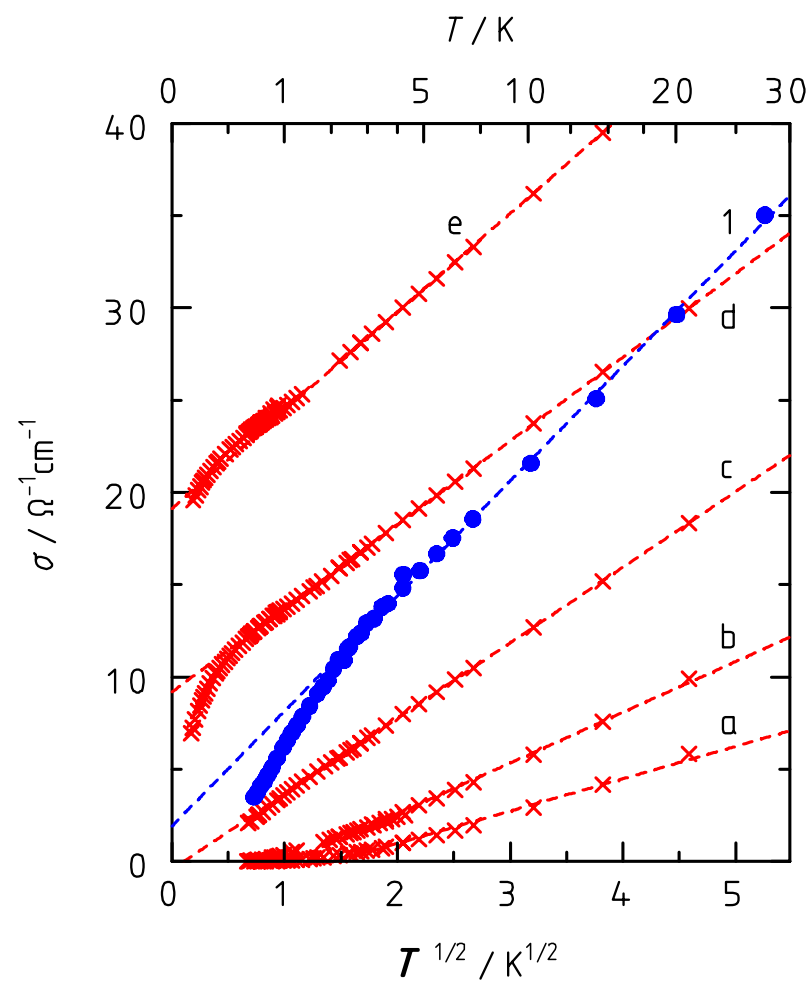
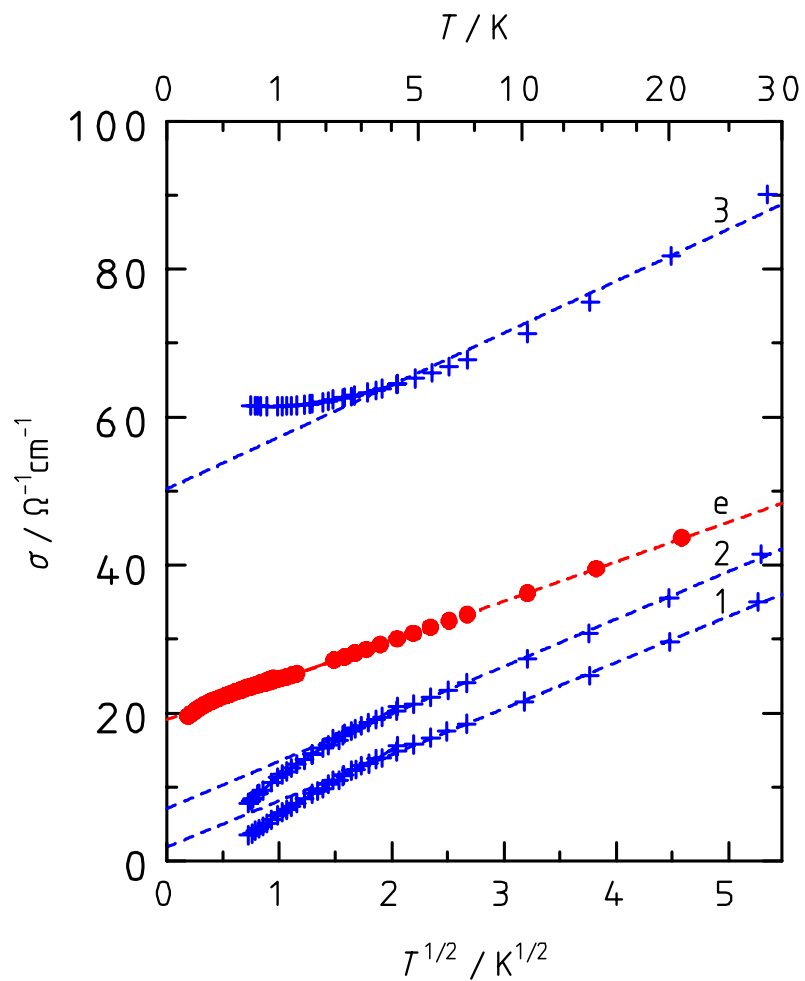
Variation of deposition conditions influences $\sigma(T)$ at low and high T in different manner.

Take home:

Different influence of preparation conditions at low and high T .

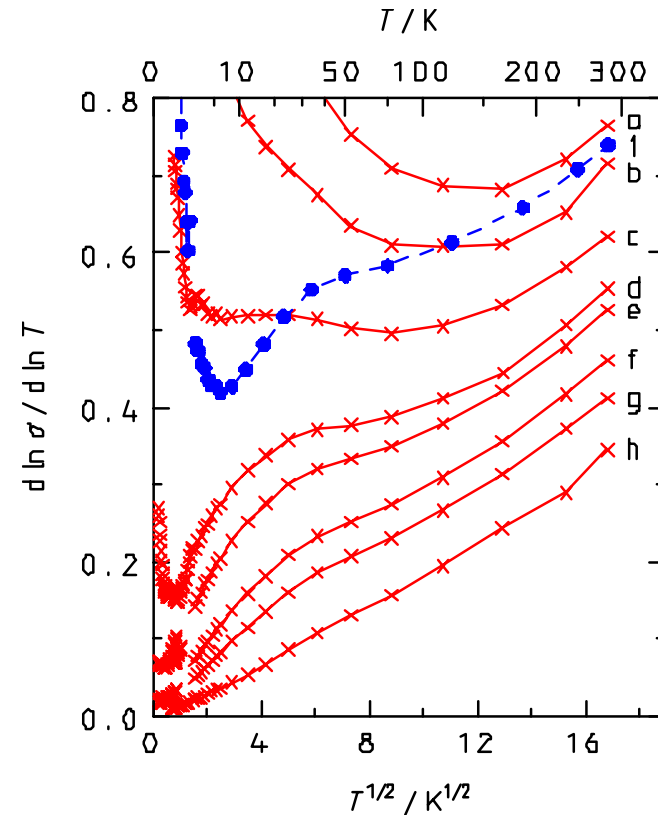
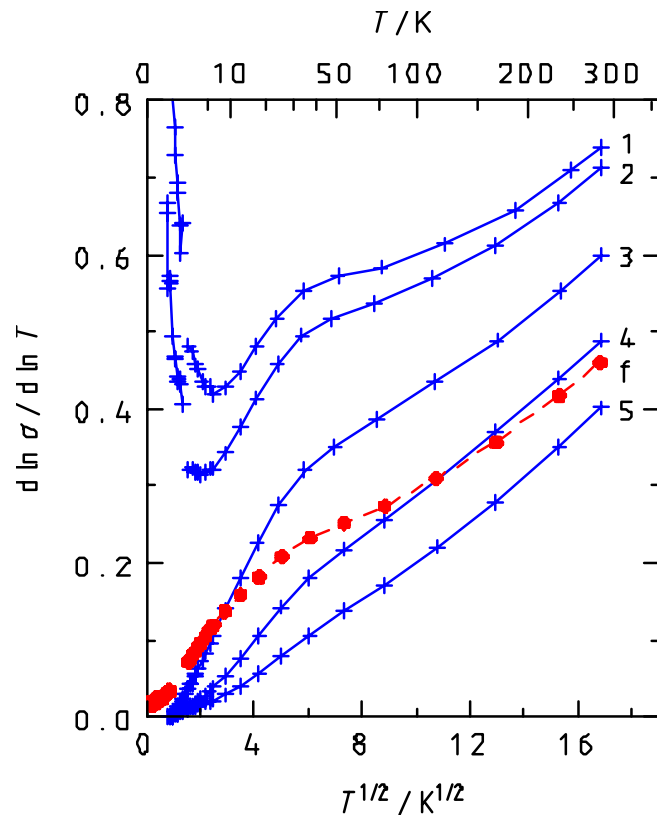
⇒ At least two conduction mechanisms should be relevant.

'Standard' extrapolation $\sigma = a + bT^{1/2}$



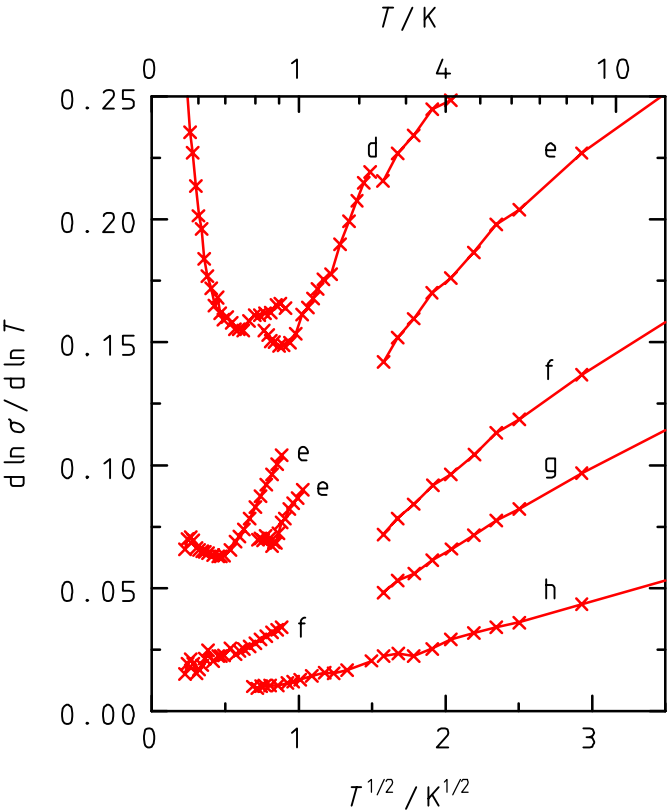
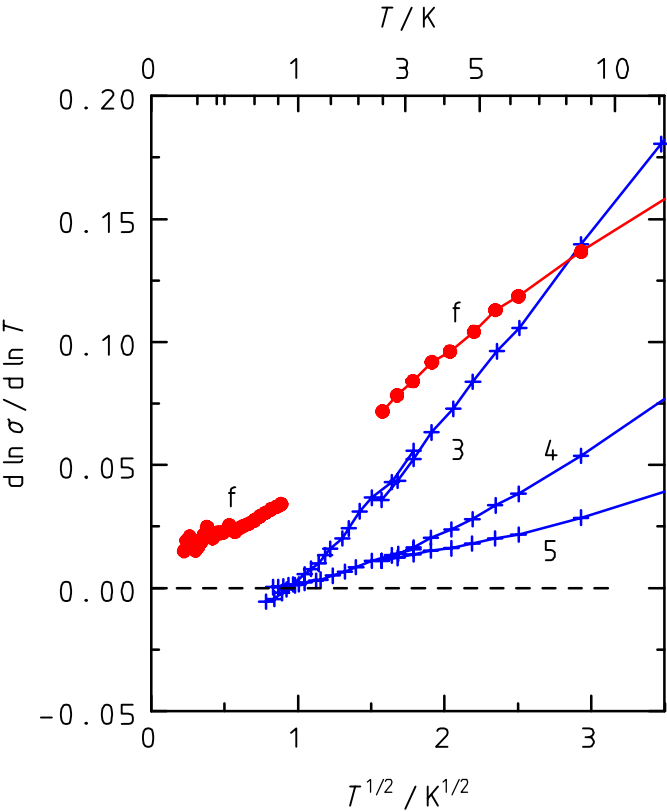
Good approximation between 2 and 30 K. Not justified for smaller T .

Logarithmic derivative



Typical: minima, probably with $T_{\min} \rightarrow 0$ and $w_{\min} \rightarrow 0$ for $x \rightarrow x_c$

Low-temperature details:



Dresden metallic samples exhibit T range with **negative $d\sigma/dT$ values**.

Discussion

Presume according to scaling theory,

$$\sigma(T=0, x) \propto (x-x_c)^{\nu} ,$$

as well as 'usual' temperature dependence,

$$\sigma(T, x) = a(x) + b(x) \cdot T^p ,$$

for $T \rightarrow 0$ (Theory: $p=1/2$ or $1/3$).

\Rightarrow Plateau is expected,

$$w(T, x_c) = p ,$$

but is missing in experiment. = severe contradiction

Two qualitative models

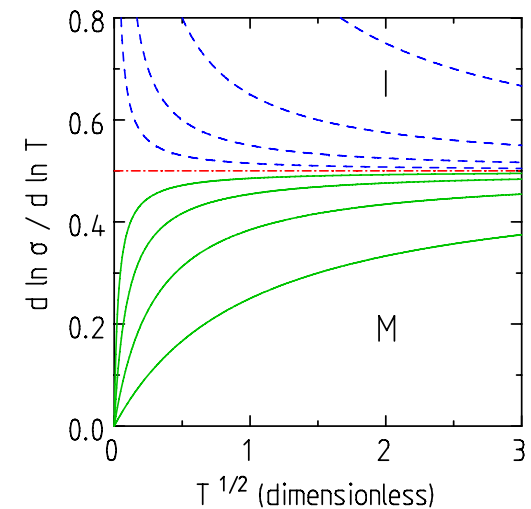
To be described:

- Continuity at transition for finite T ,
- 'usual' temperature dependences in the metallic as well as in the activated regions

A) Model of a continuous transition

$$\sigma(T, x) = \begin{cases} T^{1/2} \cdot \exp\{-(T_0(x)/T)^{1/2}\} & x < x_c \\ a(x) + T^{1/2} & x \geq x_c \end{cases} \quad \text{for } ,$$

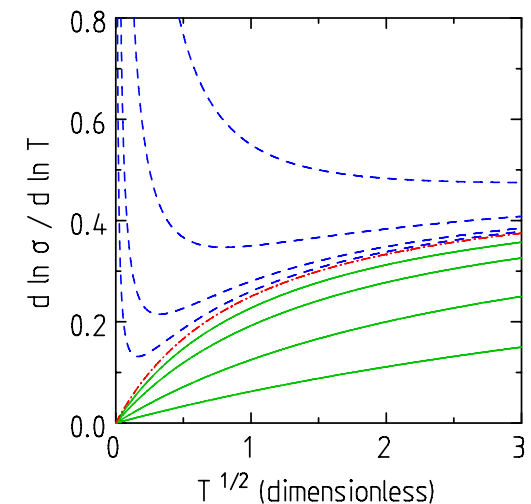
where $T_0(x \rightarrow x_c - 0) = 0$ and $a(x \rightarrow x_c + 0) = 0$.



B) Model of a discontinuous transition (discontinuous only for $T = 0$, continuous for all $T > 0$)

$$\sigma(T, x) = \begin{cases} (1 + T^{1/2}) \cdot \exp\{-(T_0(x)/T)^{1/2}\} & x < x_c \\ a(x) + T^{1/2} & x \geq x_c \end{cases} \quad \text{for } ,$$

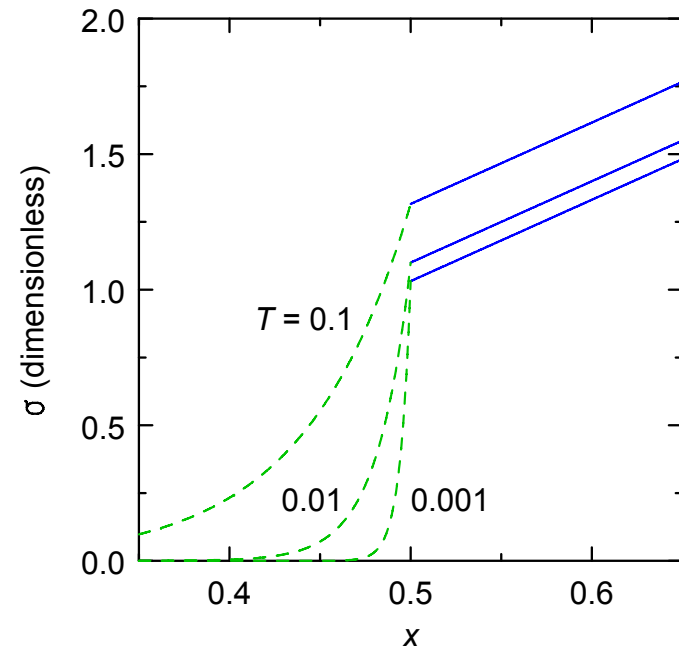
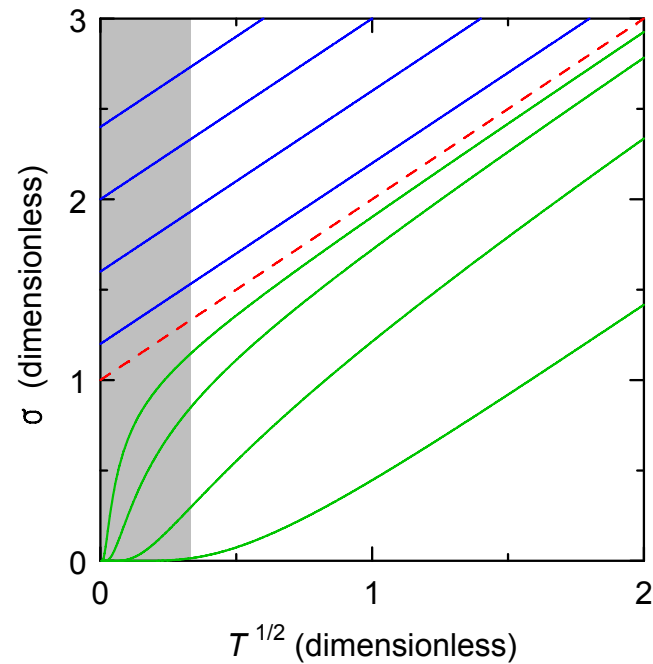
where $T_0(x \rightarrow x_c - 0) = 0$ and $a(x \rightarrow x_c + 0) = 1$.



Obvious similarity between experiment and model B

⇒ $\sigma(T=0, x)$ is discontinuous at x_c .

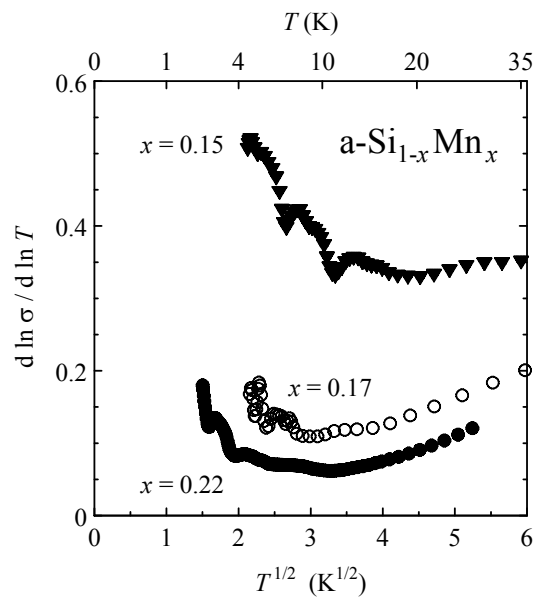
In contradiction to experience ? No!



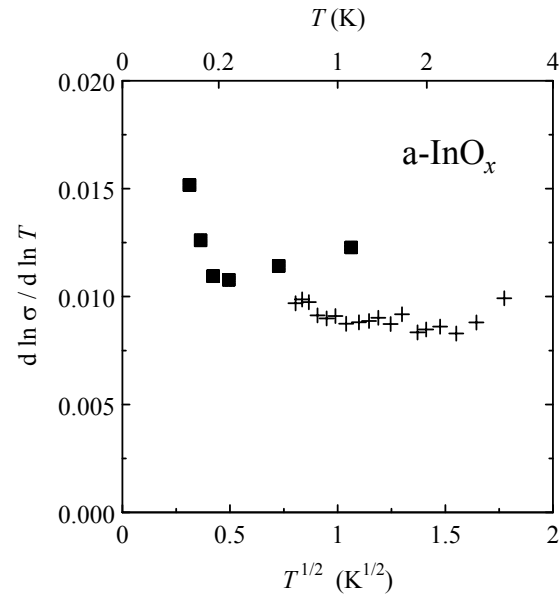
Take home:

**Deep minima in $w(T) = d \ln \sigma / d \ln T$ with $w \ll 0.5$
support existence of minimum metallic conductivity.**

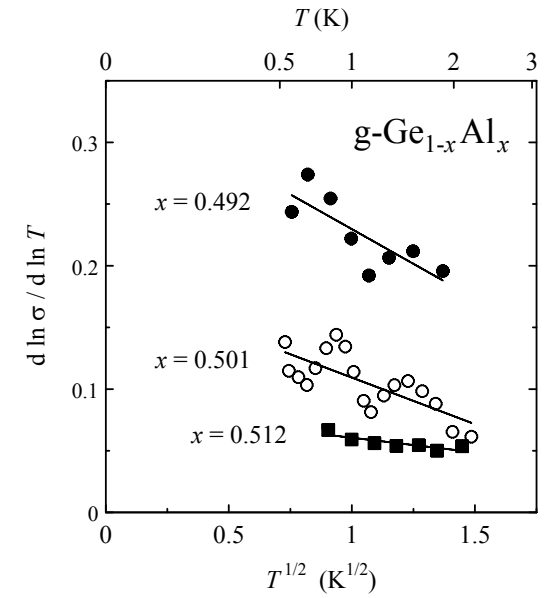
Other systems: Amorphous $\text{Si}_{1-x}\text{Ni}_x$ is not an exception



Data by Yakimov et al., 1998



Data by Graham et al., 1998



Data by Rosenbaum et al., 1994

Common feature:

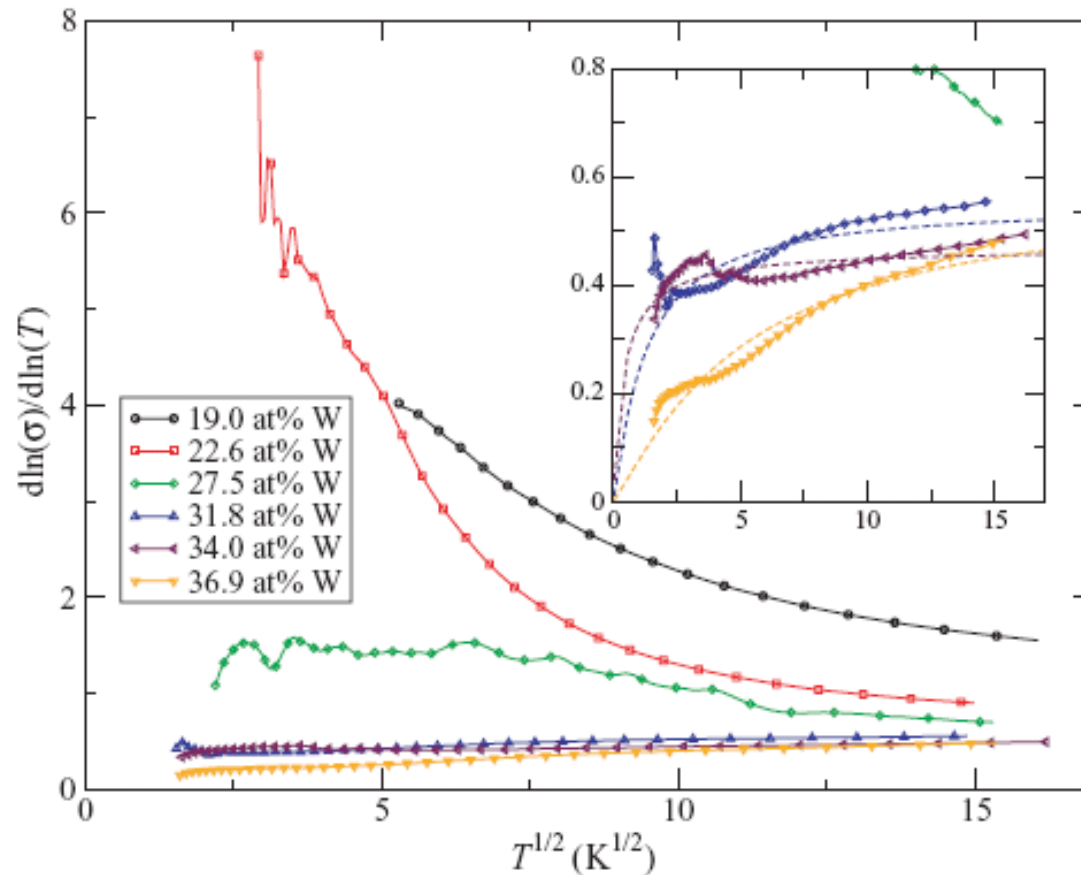
Increase of $w(T) = d \ln \sigma / d \ln T$ with decreasing T

although the values of $w(T)$ are considerably smaller than 1/2,

AM & C.J. Adkins, Current Opinion in Solid State & Materials Science **4** (1999) 303.

Current puzzle: What happens in W-based granular metals?

M. Huth, D. Klingenberger, Ch. Grimm, F. Porrati and R. Sachser,
New Journal of Physics **11**, 033032 (2009), Fig. 5:



Disprove or support of
 $\text{Si}_{1-x}\text{Ni}_x$ findings?

Contradicting hints =>
decision only by improving of
reproducibility and precision

4. Possible causes of the MIT in three-dimensional systems

Unlikely:

- **Anderson-transition of type I**, i.e. Fermi energy passing through mobility edge, cannot explain $d\sigma / dT = 0$ at the MIT for $T \rightarrow 0$.
- **Anderson transition of type II**, where small region of extended state is comprised and disappears at the MIT, requires special degree of band filling.
- **Mott-Hubbard transition**, i.e. splitting of a band, also presumes exactly half-filled band.

Most likely:

Localization due to reduction of screening: Consider electrons in screened Coulomb potential. Bound states are formed if concentration of free electrons is so low that screening length exceeds a critical value, N.F. Mott Phil. Mag. 62, 287 (1961). Supported by values of critical concentration, P.P. Edwards and M.J. Sienko, PRB 17, 2575 (1978).

5. Excursion to a two-dimensional system: Indications for a finite-temperature transition

AM, Phys. Rev. B **77**, 205317 (2008), Physica E **42**, 1243 (2010)

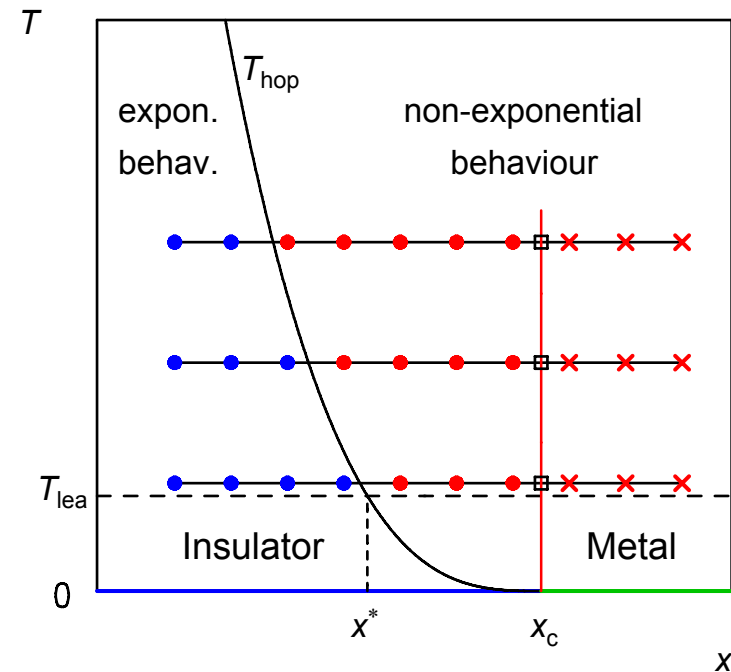
Motivation

Qualitative model for $\text{Si}_{1-x}\text{Ni}_x$ exhibits

knee in $\sigma(T = \text{const.}, x)$,

i.e., a sharp phase transition at finite T .

If such a peculiarity exists indeed, and
if it can be detected,
an unambiguous discrimination
between samples with metallic and
activated conduction will be available.



Experimental demands:

- (a) high reproducibility in sample preparation,
- (b) high degree of sample homogeneity,
- (c) high precision in chemical analysis.

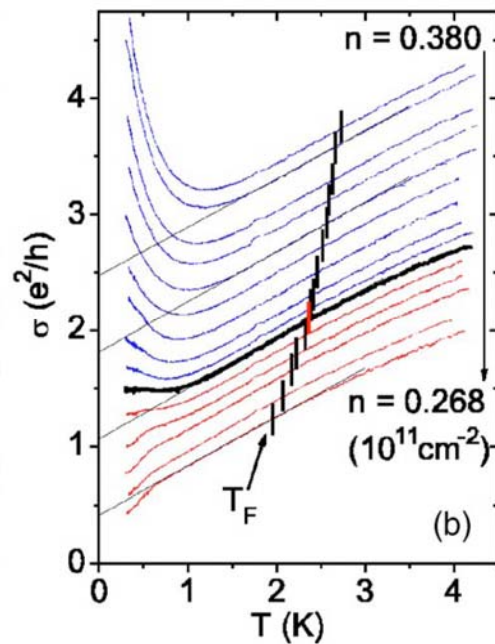
Task is simplified if MIT can be tuned by external field. This is possible in 2D systems, e.g. in *n*-type Si quantum well confined in $\text{Si}_{0.75}\text{Ge}_{0.25}/\text{Si}/\text{Si}_{0.75}\text{Ge}_{0.25}$ heterostructure:

PHYSICAL REVIEW B **75**, 033314 (2007)

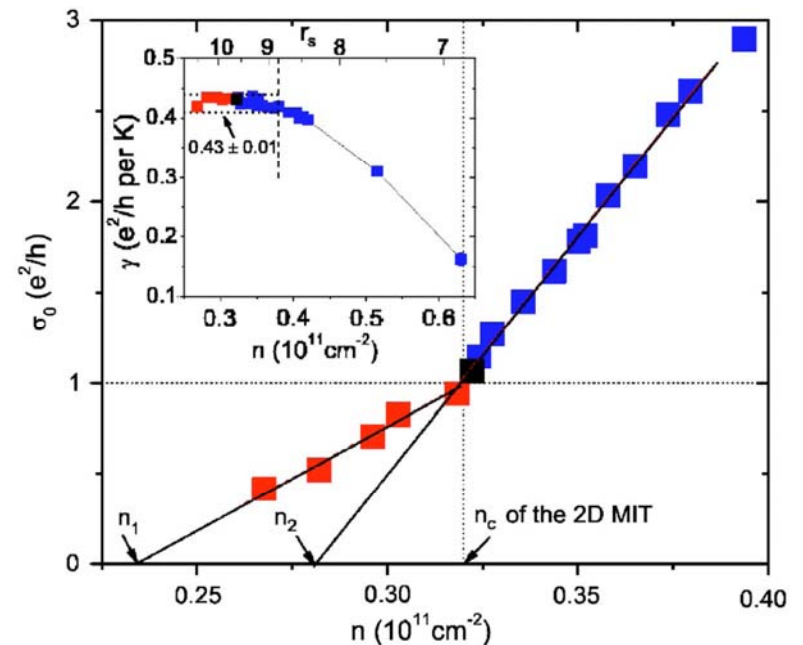
Linear temperature dependence of the conductivity in Si two-dimensional electrons near the apparent metal-to-insulator transition

K. Lai,^{1,*} W. Pan,² D. C. Tsui,¹ S. Lyon,¹ M. Mühlberger,³ and F. Schäffler³

¹Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA



extrapolation to $T = 0$



Results by Lai et al.:

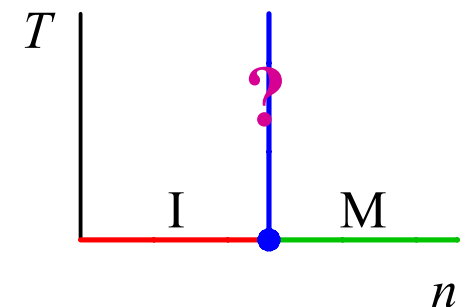
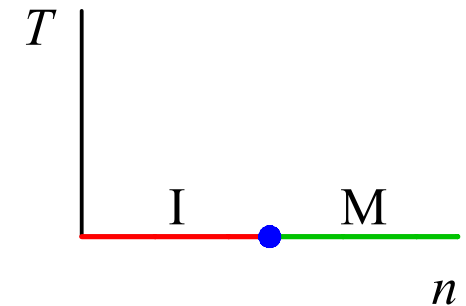
- Extrapolation $\sigma_0(n)$ exhibits sharp bend (knee), which coincides with critical concentration defined by $d\sigma / dn = 0$ at lowest T . Thus existence of different phases likely.
- Slope of $\sigma(T, n = \text{const})$ around T_F is almost constant.

Two question suggest itself:

- If $\sigma_0(n)$ has a knee, and the slope is roughly constant, should not $\sigma(T = \text{const}, n)$ exhibit a knee for finite T ?
- If yes, is its existence restricted to the regions of linear $\sigma(T, n = \text{const})$, or is it a more general phenomenon?

That means: Might there be a phase transition at finite T ?

Additionally motivated by scaling of T dependence.



Empirical situation

Compare two empirical 4-parameter fits:

Piecewise linear,

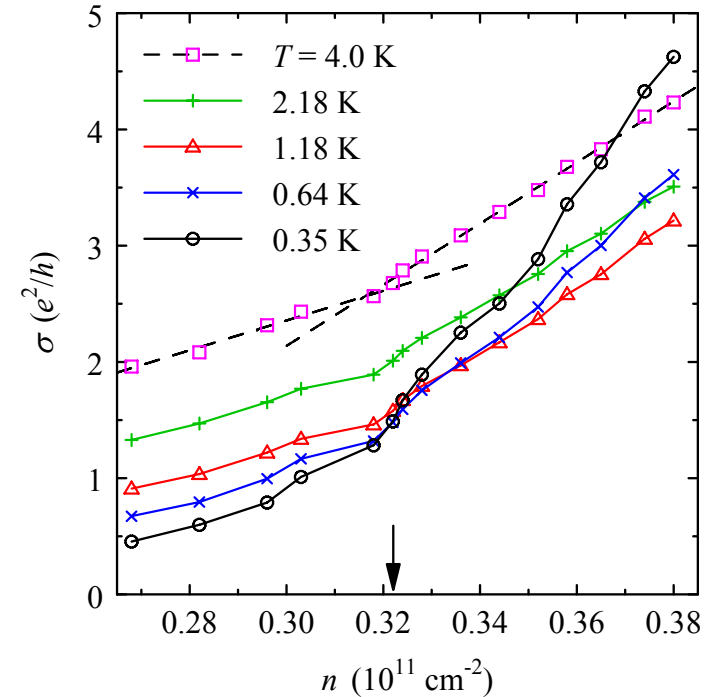
$$f_{\text{plf}} = a + b n + c (n - n_k) \theta(n - n_k) ,$$

and polynomial,

$$f_{\text{pto}} = p + q n + r n^2 + s n^3$$

Results for complete data set:

$T(\text{K})$	$n_k (10^{-11} \text{ cm}^{-2})$	$\chi_{\text{plf}}^2 (e^4/h^2)$	$\chi_{\text{pto}}^2 (e^4/h^2)$
4.0	0.318	0.010	0.024
2.18	0.318	0.006	0.019
1.18	0.318	0.010	0.019
0.64	0.318	0.028	0.031
0.35	0.315	0.097	0.063



⇒ **Clear advantage for
piecewise linear fit
at high T**

In detail

Problem of comparison: σ range broadens when $T \rightarrow 0$ and
 $\sigma(n)$ is nonlinear at insulator side (vanishes exponentially).

\Rightarrow Fixed σ range is advantageous.

Results for restricted set, $0.5 \sigma(T, n_c) < \sigma(T, n) < 2 \sigma(T, n_c)$:

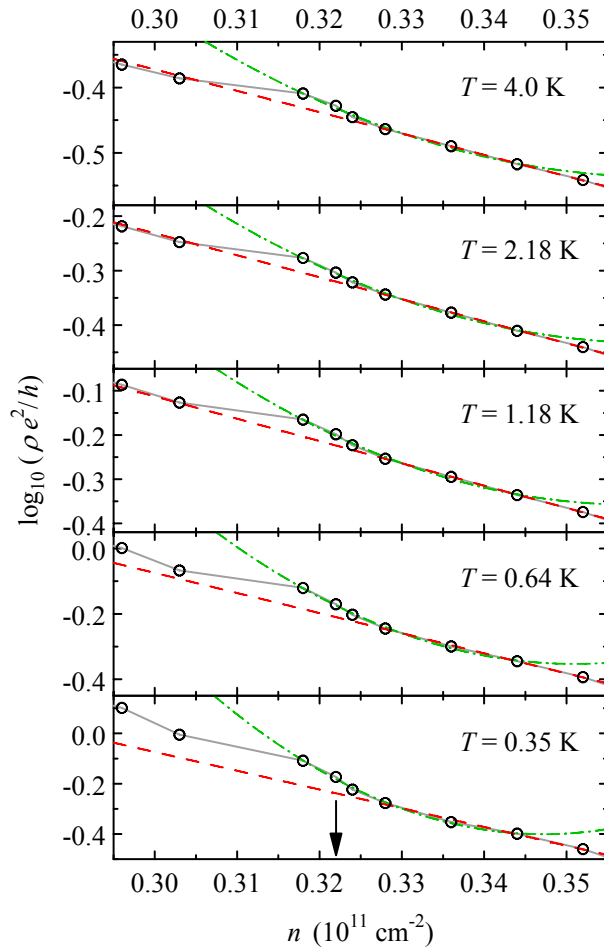
T (K)	N	n_k (10^{-11} cm^{-2})	χ_{plf}^2 (e^4/h^2)	χ_{pto}^2 (e^4/h^2)
4.0	15	0.318	0.010	0.024
2.18	15	0.318	0.006	0.019
1.18	14	0.318	0.009	0.018
0.87	13	0.318	0.009	0.020
0.64	11	0.318	0.013	0.026
0.47	9	0.318	0.023	0.032
0.35	9	0.318	0.024	0.036

\Rightarrow Piecewise linear function is always clearly better than polynomial.

\Rightarrow All knee positions very close to critical concentration from $d\sigma/dT = 0$

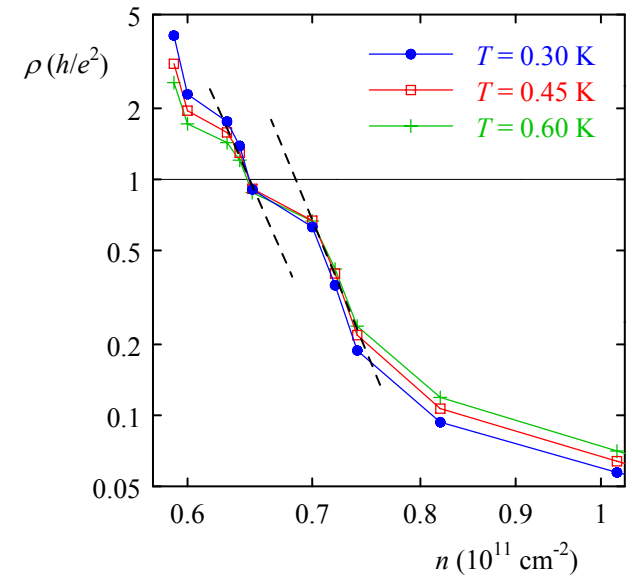
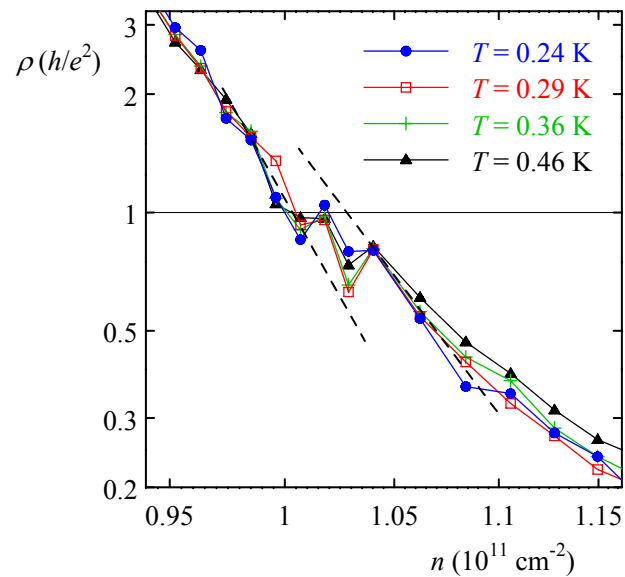
Comparison with other experiments

Consider usual log-log plots: Data by Lai et al. exhibit “rounded step” for all T .



In 2002, similar structures were found in data for high mobility MOSFET by Kravchenko et al. (1995)

AIAs quantum well by Papadakis and Shayegan (1998)



Significance

Peculiarities might arise from random deviations, but it is very unlikely that all following coincidences occur only by chance:

- (a) Observed at **three independent experiments** at different materials.
- (b) Peculiarities have **qualitatively same form** in all three cases.
- (c) Always in **same resistivity region**, slightly below h / e^2 .
- (d) Always **close to common intersection point** of $\rho(T = \text{const}, n)$ curves.
- (e) Features seem to be present for **all considered T** .

Not found in further experiments, but may be easily overlooked due to inhomogeneities or too small density of measuring points (concerning n).

Scaling analysis

Suppose, $\sigma(T, n)$ scales as found at Si MOSFETs by Kravchenko et al.:

$$\sigma(T, n) = \sigma(t) \quad \text{with} \quad t = T/T_0(n)$$

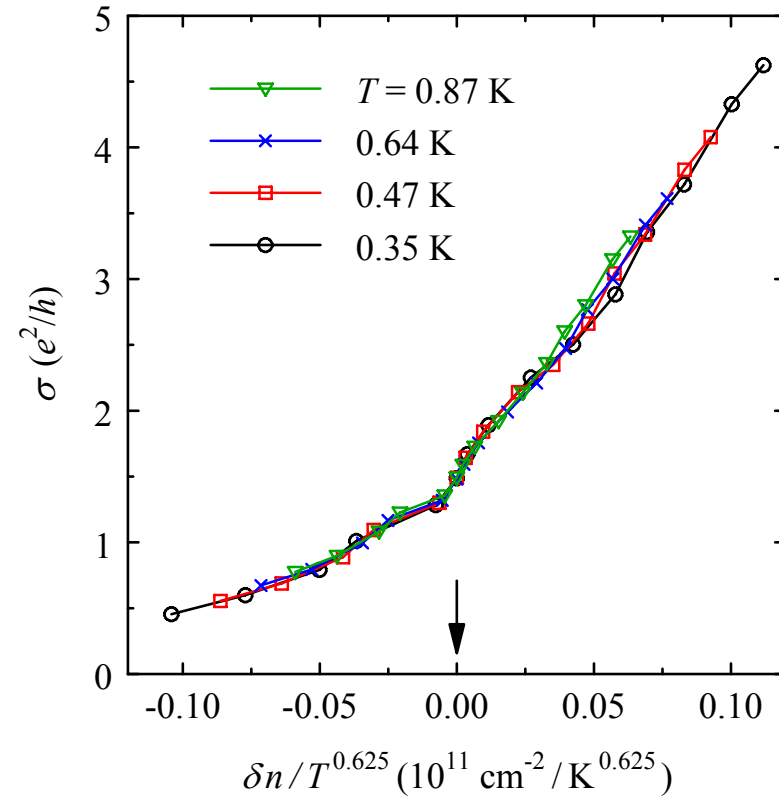
Assume, scaling holds up to n_c , where T_0 vanishes, and

$$T_0(n) = A |\delta n|^\beta \quad \text{with} \quad \delta n = n - n_c.$$

Thus, σ should depend only on $T/|\delta n|^\beta$ or rather on $\delta n/T^{1/\beta}$.

Exponent β should be universal,

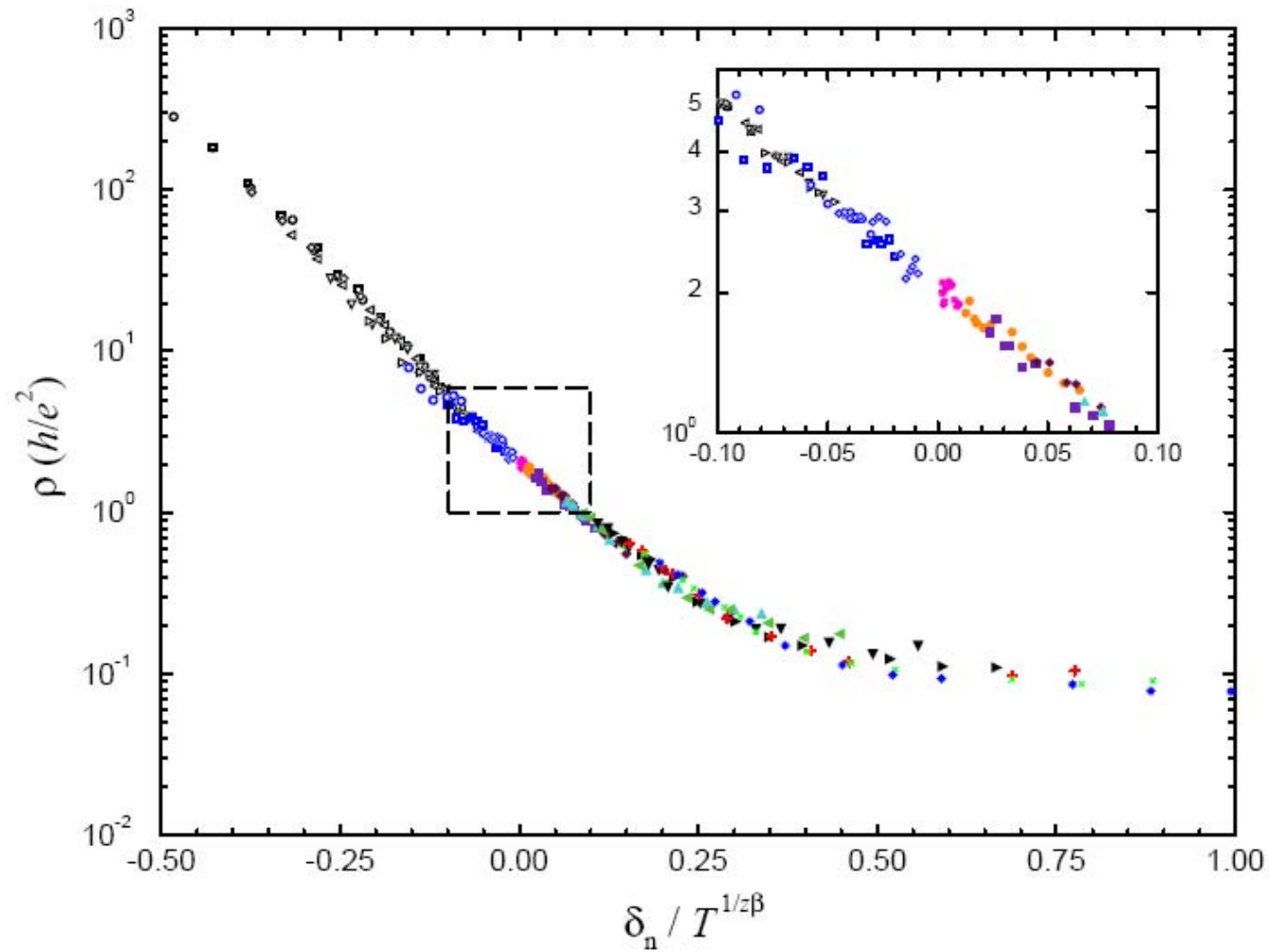
$\beta = 1.6 \pm 0.1$ for Si MOSFET.



⇒ **Scaling check without fit passed**

Scaling from previous work

$\rho(T,n)$ instead of $\sigma(T,n)$ for Si-MOSFET by Kravchenko et al., 1995:



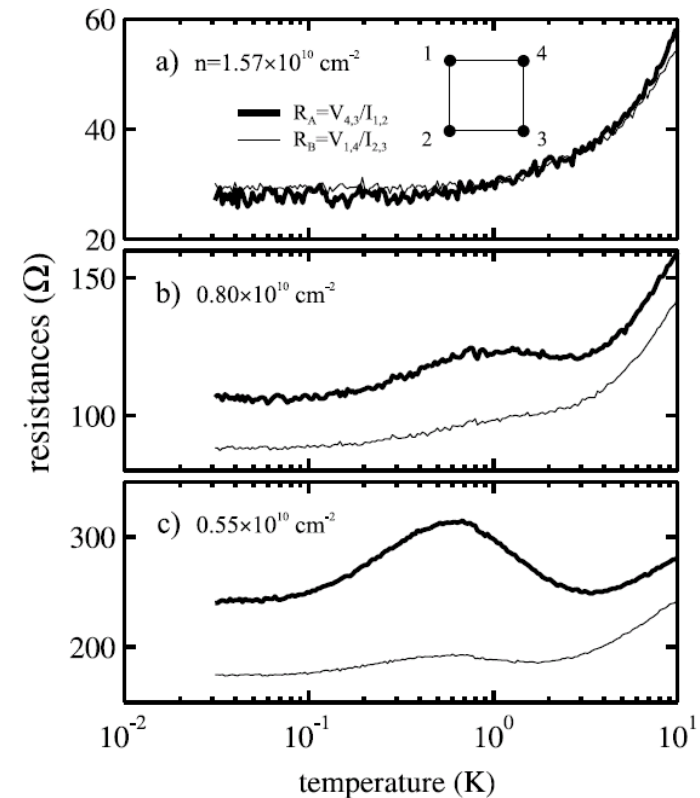
Scaling: **yes**

Knee: **no**,
but broader scatter
of data points

⇒ Question: Why is “knee” present in experiment by Lai et al., but not in Kravchenko’s study although critical exponent seems to be the same?

Possible reasons:

- Peculiarity is rather small so that it can easily be overseen on logarithmic scale.
- Already sample inhomogeneities of a few percent may wash out such a structure, while, far from the transition, they yield only a second order contribution



Van der Pauw study of GaAs heterostructure by M.P. Lilly et al., PRL **90**, 056806 (2003)

Remark

Data collapse **without parameter adjustment** is quite convincing, but contains hidden message:

Motivation for scaling analysis from own experience for three-dimensional amorphous $\text{Si}_{1-x}\text{Cr}_x$ films.

There: scaling only in hopping region. Reason for non-scaling at metallic side: x -dependence of $\sigma(T = 0, x)$.

Here, for $d = 2$, hint to scaling at both sides of MIT as in Kravchenko's MOSFET study.

This would imply **new phase**.

Why?

In case of **conventional metallic conduction**, as $T \rightarrow 0$:

$\sigma(T, n) \rightarrow \text{finite } \sigma(0, n)$, which increases monotonously with n .

Vanishing T corresponds to diverging $\delta n / T^{0.625}$.

Thus, curves drawn in scaling plot for varying T and fixed δn should split,

in contradiction to observed scaling.

\Rightarrow Scaling for $n > n_c$ cannot be understood in terms of conventional metallic conduction.

“Superconductivity” (only) at $T = 0$ may be an alternative, comp. Kravchenko et al. 1995.

\Rightarrow Data collapse at metallic side = **big puzzle**

However, saturation at n dependent σ values as $T \rightarrow 0$ in many other experiments.

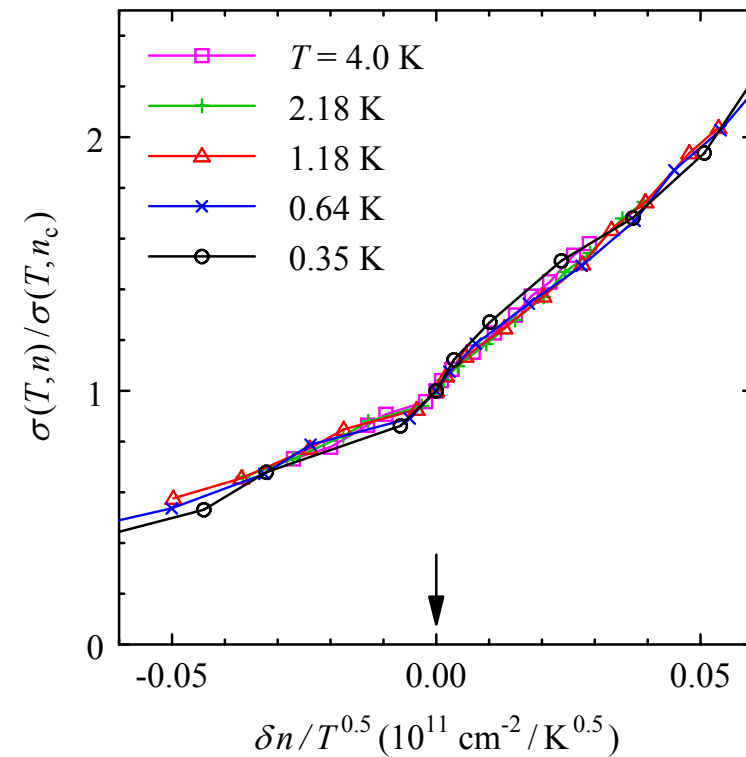
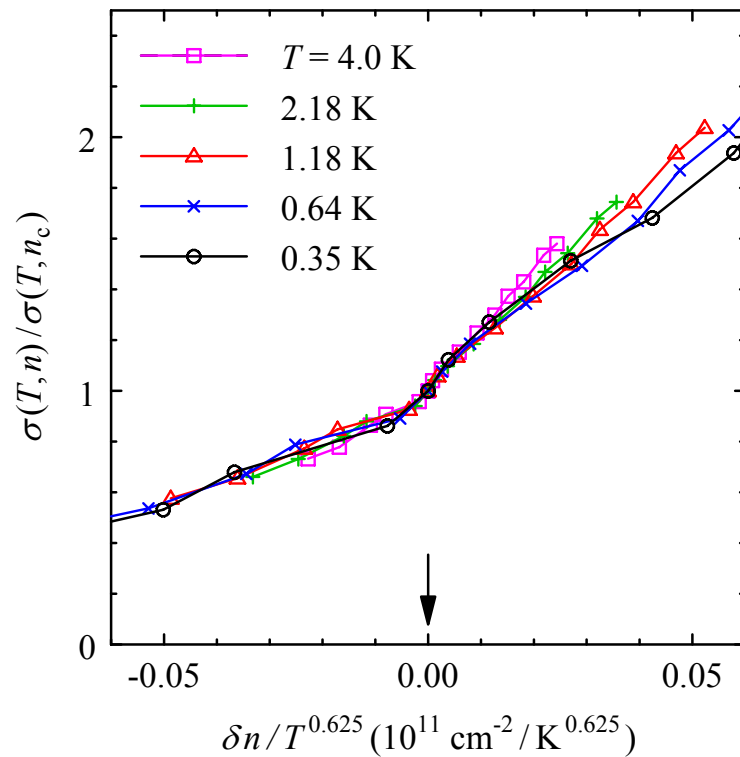
\Rightarrow Questions for existence range of data collapse, and for relevance of thermal decoupling.

Generalisation to $T > 1$ K

Let T increase: Scaling $\sigma(T, n) = \sigma(t)$ breaks down when separatrix $\sigma(T, n_c)$ becomes T -dependent. Experience from 3D systems suggests hypothesis

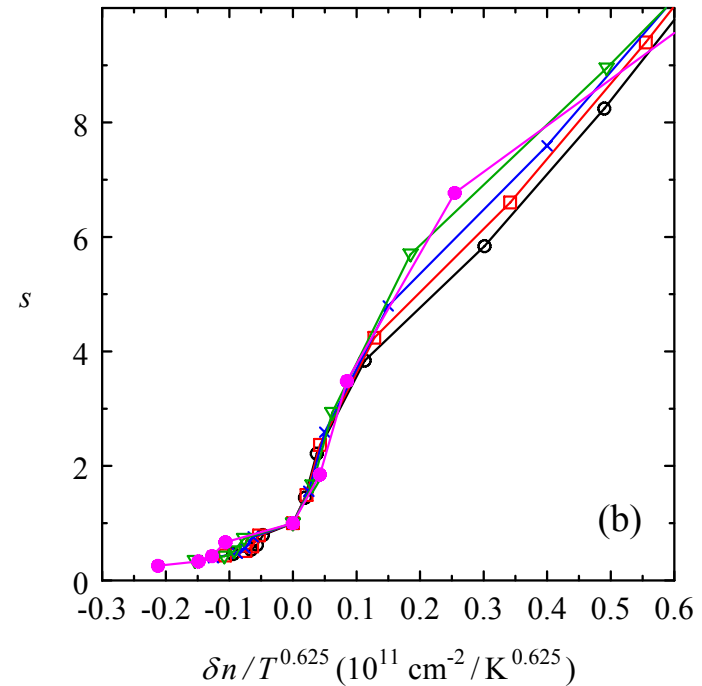
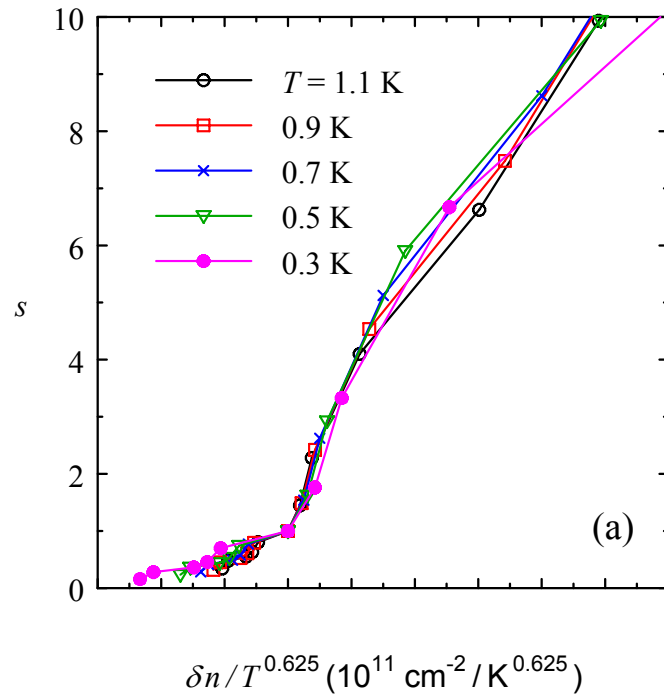
$$\sigma(T, n) = \sigma_{\text{scal}}(T/T_0(n)) \cdot \xi(T/T_1) \quad \text{with } T_1 = \text{const.}$$

$\Rightarrow s(T, n) = \sigma(T, n) / \sigma(T, n_c)$ may be scaleable. Tests for $\beta = 1.6$ and 2:



Comparison to AIAs

Turn once more to data by Papadakis and Shayegan (1998): $s(T,n) = \sigma(T,n) / \sigma(T,n_c)$



Hidden problem: slight uncertainty of critical concentration from measurements for orthogonal directions (a) and (b). Thus, presume $n_c = 0.70 \cdot 10^{11} \text{ cm}^{-2}$.

Peculiarity at MIT

Possible explanation of “indentation” in $\sigma(T = \text{const}, n)$ around MIT:

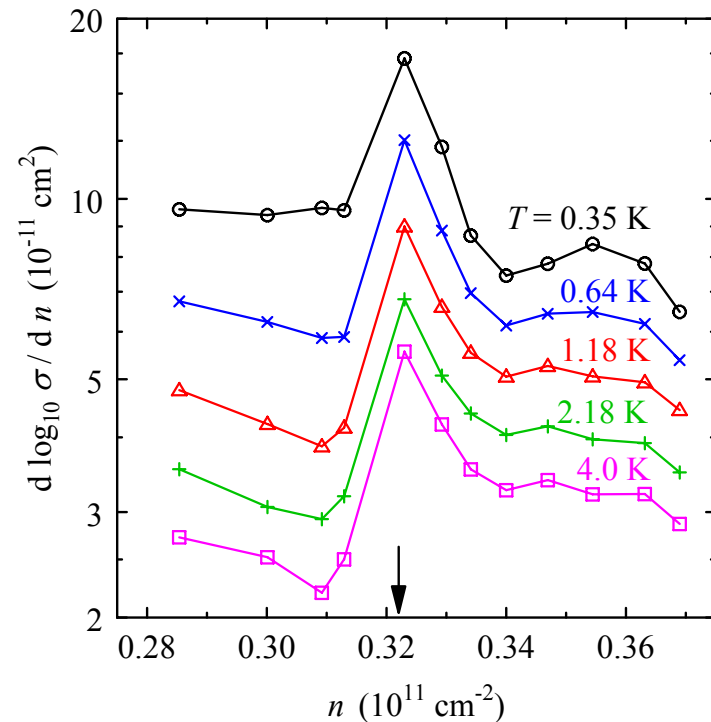
Consider $n \rightarrow n_c - 0$, where $T_0 \rightarrow 0$
 so that $t = T/T_0 \rightarrow \infty$ for all T . Suppose,

$$\sigma(t) = \sigma_c \cdot (1 - B t^{-\nu})$$

with $0 < \nu < 1/2$ (hopping). Thus,

$$\sigma = \sigma_c \cdot (1 - C T^{-\nu} |\delta n|^{\nu\beta})$$

with $\nu\beta < 1$, so that $d\sigma/dn$ and $d \log_{10} \sigma / dn$ should diverge.



⇒ **Sharp peaks present for all T**
 (logarithmic scale of ordinate)

For previous analysis of $d\sigma/dn(n)$ see A.M. Mack et al., arXiv:cond-mat/9807040.

Take home:

There is a series of indications for a line of phase transitions at finite T connected with the apparent MIT at $T = 0$:

Advantage of piecewise linear fit compared to power law approximation

Knee position close to n_c and independent of T

Scaling analysis leads to data collapse without fit

Similarity to two previous experiments concerning offset in log-log plots

Sharp peaks in $d \log_{10} \sigma / d n$ as function of n for all T values considered

⇒ **Challenges** to colleagues:

? Check by independent experiment

? Nature of apparently metallic phase

6. Conclusions & challenges

- Metal-insulator transition in disorder systems is still a fascinating field.
- **Precision matters.**
- Logarithmic derivative yields much evidence for a discontinuity of the MIT at $T = 0$ in three-dimensional samples: \Rightarrow **Mott was probably right.**
- In two-dimensional systems, this discontinuity may be connected with a continuous transition at finite T
- ? Extension to other materials
- ? Can MIT be detected at finite T in three-dimensional systems?
- ? Magnetotransport
- ? Theoretical explanation of behaviour of logarithmic derivative

Appendix: Exact analysis

Presumptions:

1. $\sigma(T = T_m, x)$ measured.
2. Monotonicity of $\sigma(T = \text{const.}, x)$.
3. Classification according to $w(T, x)$:

‘Activated’ samples have individual lower bounds of $w(T)$ for *each* of the samples: There is a $w_a(x) > 0$ such that $w(T, x) > w_a(x)$ for all $T \leq T_m$.

‘Metallic’ samples have individual upper bounds of $w(T)$:

There are $w_m(x)$ and q with $w(T, x) < w_m(x) \cdot (T/T_m)^q$ for $T \leq T_m$.

4. Experiment suggests that $w_m(x)$ is bounded from above and that $q > 0$ and finite :

There is a w_0 with $w_m(x) \leq w_0$ for $x \geq x_c$. \iff Separatrix $w(T, x_c) \rightarrow 0$ for $T \rightarrow 0$.

Bounds for $\sigma(T < T_m, x)$ by means of integration:

'Activated' sample:

$$w_a(x) < w(T, x)$$

$$w_a(x) \cdot (\ln T_m - \ln T) < \ln \sigma(T_m, x) - \ln \sigma(T, x)$$

$$\sigma(T, x) < \sigma(T_m, x) \cdot (T/T_m)^{w_a(x)}$$

$$\lim_{T \rightarrow 0} \sigma(T, x) = 0$$

'Metallic' sample:

$$w_m(x) \cdot (T/T_m)^q > w(T, x)$$

Due to $dT^q / d \ln T = q \cdot T^q$:

$$w_m(x) \cdot (1 - (T/T_m)^q) / q > \ln \sigma(T_m, x) - \ln \sigma(T, x)$$

$$\sigma(0, x) > \sigma(T_m, x) \cdot \exp\{-w_m(x)/q\} > 0$$

Consequences:

1. Classifications according to $w(T, x)$ and to $\sigma(T, x)$ are identical.
2. For all metallic samples, due to the monotonicity of $\sigma(T=const., x)$ and $w_m(x) \leq w_0$:

$$\sigma(0, x) \geq \sigma(0, x_c) > \sigma(T_m, x_c) \cdot \exp\{-w_0/q\}$$

⇒ There is a finite minimum metallic conductivity,

$$\sigma_{\min \text{ met}} = \sigma(0, x_c) > \sigma(T_m, x_c) \cdot \exp\{-w_0/q\}$$

Numbers: $T_m = 1 \text{ K}$, $q = .5$, $w_0 = .05 \Rightarrow \exp\{-w_0/q\} = .9$

Dresden: $x_c = 15.9 \pm 1 \text{ at\% Ni}$, $20 \Omega^{-1}\text{cm}^{-1} < \sigma_{\min \text{ met}} < 65 \Omega^{-1}\text{cm}^{-1}$,

Tel Aviv: $x_c = 17 \pm 3 \text{ at\% Ni}$, $25 \Omega^{-1}\text{cm}^{-1} < \sigma_{\min \text{ met}} < 70 \Omega^{-1}\text{cm}^{-1}$.

$\sigma_{\min \text{ met}}$ data by roughly factor of 2 to 5 smaller than theoretical value. Possible reasons: uncertain constants, d character of states, clusters.