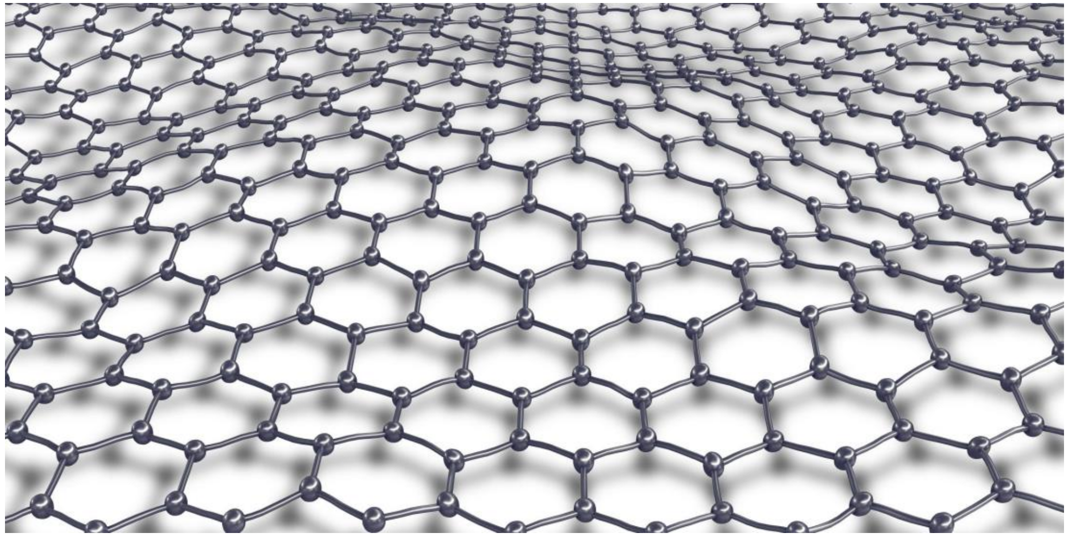
Mesoscopic Physics in Carbon Based Devices

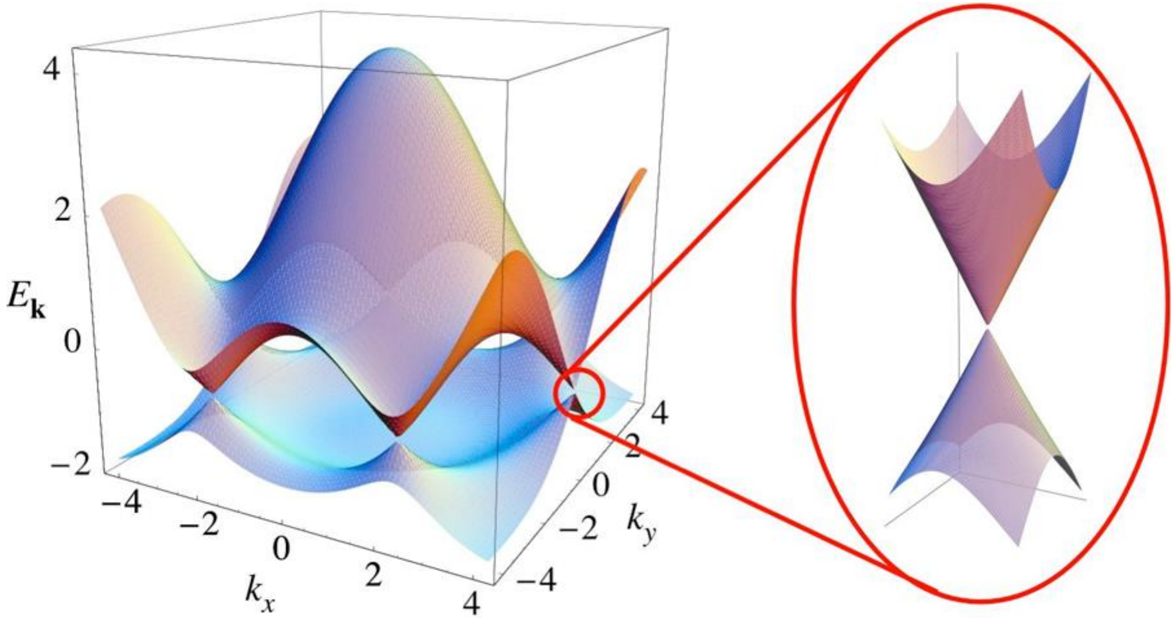
I/ Mesoscopic Physics and Quantum Transport in Graphene

The fundamental properties of graphene (transport)

Discovered in 2004 by Geim and Novoselov who received the Nobel Prize in 2010

- <u>A perfect 2D metal</u>: single sheet of atoms!
- Band Structure: <u>a semi-metal with relativistic particles</u>.





The valence band and the conduction band touch each other at two non-equivalent points of the Brillouin's zone (K & K').

→Dirac points

Close to K and K': the dispersion relation is linear

$$E = \pm\hbar v_F \Delta k == \pm\hbar v_F q$$

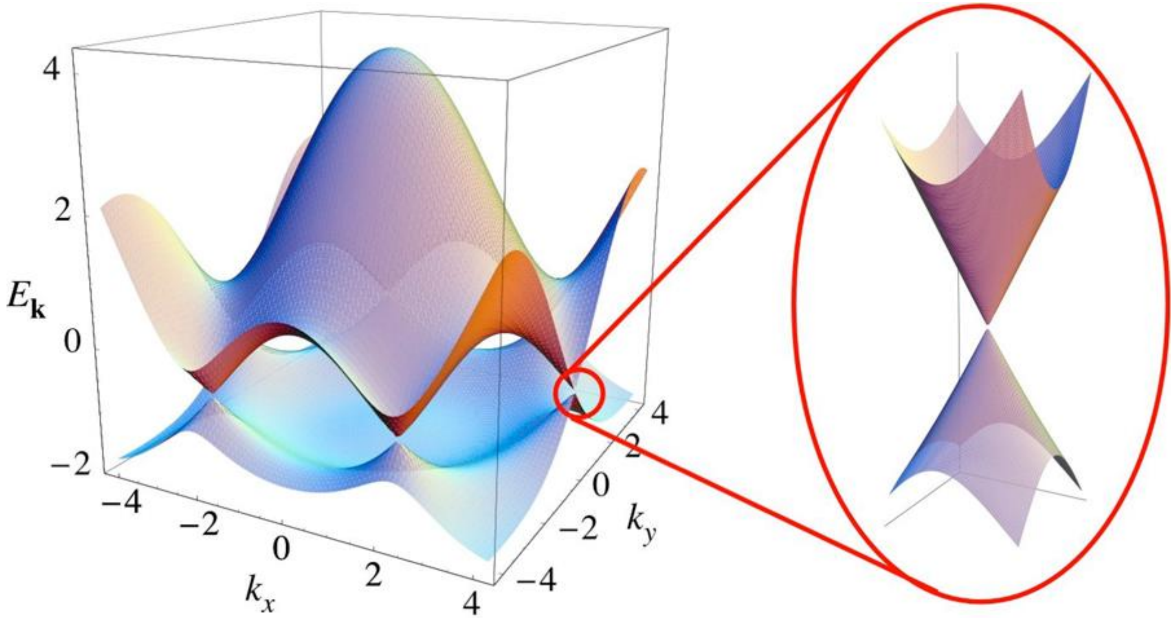
($\pm$ stands for electrons or holes)

This dispersion relation is typical of massless relativistic particles (for non-relativistic particles, $E \propto q^2$):

relativistic physics → $E^2 = m^2c^4 + p^2c^2$

(p is the momentum of the particle $p = \hbar k$)

Two different limits can be considered…



a. The case of a massive particle ($m \gg \frac{p}{c}$)

In this case, the energy can be approximate by:

$$E = \sqrt{m^2c^4 + p^2c^2}$$

$$E = mc^2\left(1 + \frac{1}{2}\frac{p^2}{m^2c^2}\right)$$

$E = mc^2 + \frac{p^2}{2m}$As mc^2 is constant, we recover the usual quadratic dispersion relation $E \propto q^2$ of a massive particle.

a. The case of a massive particle ($m \gg \frac{p}{c}$)

In this case, the energy can be approximate by:

$$E = \sqrt{m^2c^4 + p^2c^2}$$
$$E = mc^2\left(1 + \frac{1}{2}\frac{p^2}{m^2c^2}\right)$$

$E = mc^2 + \frac{p^2}{2m}$As mc^2 is constant, we recover the usual quadratic dispersion relation $E \propto q^2$ of a massive particle.

b. The case of a relativistic massless particle ($m \ll \frac{p}{c}$)

In this case, the energy can be approximate by:

$$E = \sqrt{m^2c^4 + p^2c^2}$$
$$E = pc\left(1 + \frac{1}{2}\frac{m^2c^2}{p^2}\right)$$
$$E = c\hbar k$$

At the smallest order in mc/p, the dispersion relation is linear. Like for photons.
Fundamental differences:

- charged particles
- Fermions
- the “speed of light” is about 10^6 m/s instead of 3.10^8 m/s for photons.

- **Very high mobility**.
mobility μ: drift velocity of the electron $\vec{v} = \mu\vec{E}$
conductivity $\sigma = en\mu \;\Leftrightarrow\; \mu = e\tau/m$

Metals	Semi-conductors (2DEG)	Semi-conductors (bulk)
σ=10^{7} S/m	σ=3.10^{-2} S/□	σ=10^{-6} -10^{4} S/m
n=10^{29} m^{-3}	n=1-2.10^{11} cm^{-2}	n=10^{15}-10^{20} cm^{-3}
μ=6.10^{-4} m^2/Vs	μ up to 3,5.10^{3} m^2/Vs @4K	μ=10^{-2} m^2/Vs

mobility of graphene: up to 200 000 cm^2/Vs at liquid helium temperature!

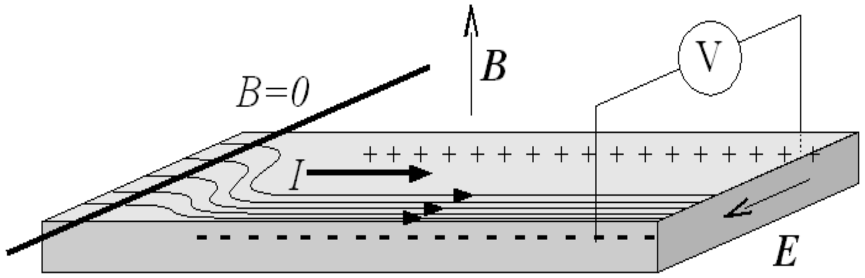
- Band structure: **ambipolar behavior**
changing the position of E_F thanks to a gate voltage allows us to measure either electrons or holes.

II/ Magnetotransport in 2D-metals: from the Hall effect to the quantum Hall effect

1. Classical Hall effect

Dirty 2D-metal embedded in a low magnetic field. Deviation of the charge carriers due to the Lorenz force: $\overrightarrow{F_L} = q\vec{v} \wedge \vec{B} = \frac{\vec{j}}{n} \wedge \vec{B}$.

→ Charge non-equilibrium that induces a transverse electric field $\overrightarrow{E_H}$ which counteracts the effect of the Lorenz Force once the steady state is reached: $q\overrightarrow{E_H} = \overrightarrow{F_H} = -\overrightarrow{F_L} = -q\vec{\nabla}V \;\Rightarrow\; \vec{\nabla}V = \frac{\vec{j}}{qn} \wedge \vec{B}$.



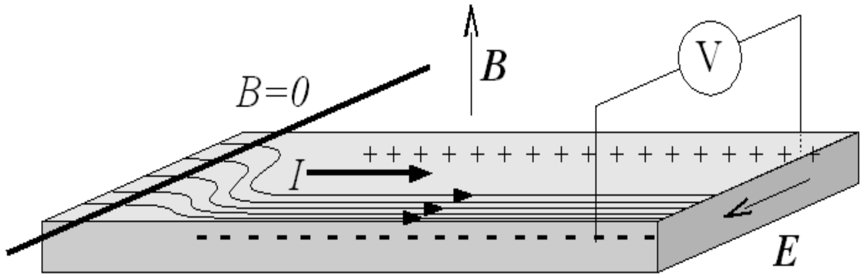
II/ Magnetotransport in 2D-metals: from the Hall effect to the quantum Hall effect

1. Classical Hall effect

Dirty 2D-metal embedded in a low magnetic field. Deviation of the charge carriers due to the Lorenz force: $\overrightarrow{F_L} = q\vec{v} \wedge \vec{B} = \frac{\vec{j}}{n} \wedge \vec{B}$.

→ Charge non-equilibrium that induces a transverse electric field $\overrightarrow{E_H}$ which counteracts the effect of the Lorenz Force once the steady state is reached: $q\overrightarrow{E_H} = \overrightarrow{F_H} = -\overrightarrow{F_L} = -q\vec{\nabla}V \Rightarrow \vec{\nabla}V = \frac{\vec{j}}{qn} \wedge \vec{B}$.

We have then:



$$\int \vec{\nabla}V \cdot \overrightarrow{dl} = \int \frac{\vec{j}}{qn} \wedge \vec{B} \cdot \overrightarrow{dl}$$

$$V_H = -\frac{B}{qn} \int \vec{j} \cdot \overrightarrow{dl} = -\frac{BI}{qn}$$

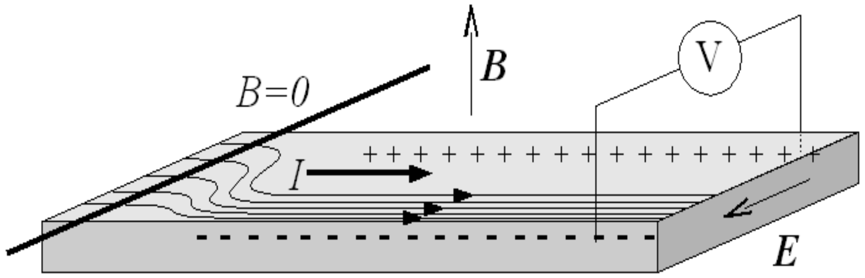
II/ Magnetotransport in 2D-metals: from the Hall effect to the quantum Hall effect

1. Classical Hall effect

Dirty 2D-metal embedded in a low magnetic field. Deviation of the charge carriers due to the Lorenz force: $\overrightarrow{F_L} = q\vec{v} \wedge \vec{B} = \frac{\vec{j}}{n} \wedge \vec{B}$.

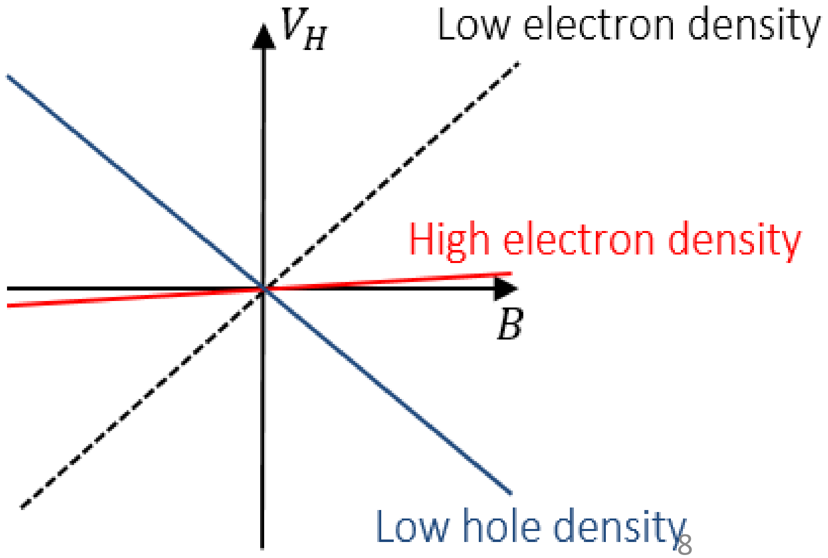
$\rightarrow$ Charge non-equilibrium that induces a transverse electric field $\overrightarrow{E_H}$ which counteracts the effect of the Lorenz Force once the steady state is reached: $q\overrightarrow{E_H} = \overrightarrow{F_H} = -\overrightarrow{F_L} = -q\vec{\nabla}V \Rightarrow \vec{\nabla}V = \frac{\vec{j}}{qn} \wedge \vec{B}$.

We have then:



$$\int \vec{\nabla}V \cdot \overrightarrow{dl} = \int \frac{\vec{j}}{qn} \wedge \vec{B} \cdot \overrightarrow{dl}$$

$$V_H = -\frac{B}{qn} \int \vec{j} \cdot \overrightarrow{dl} = -\frac{BI}{qn}$$

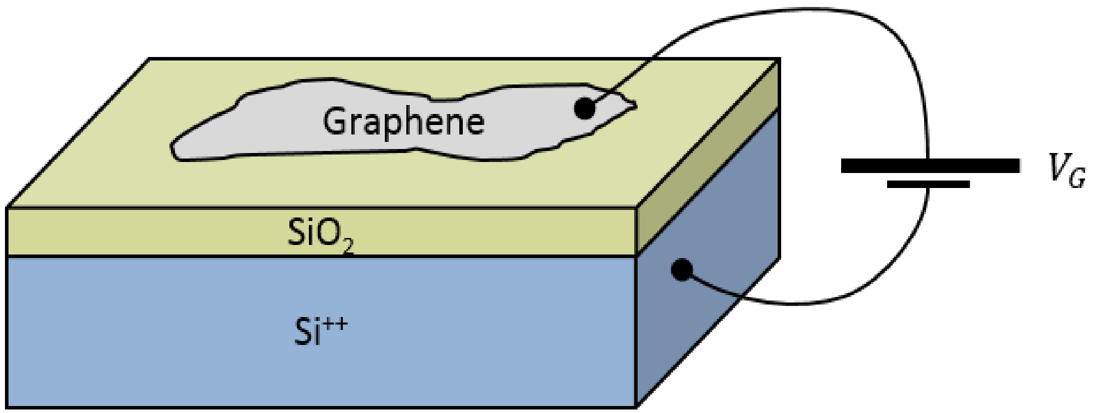


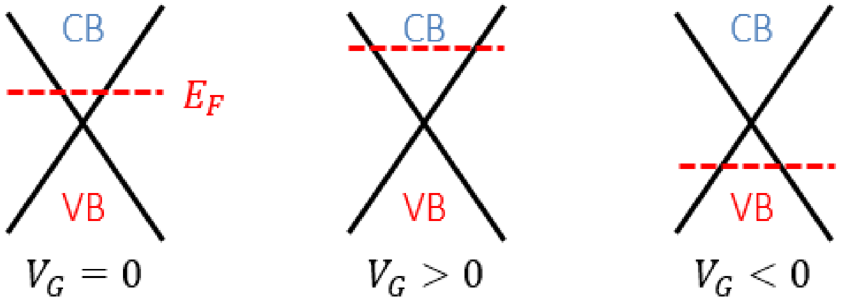
We can define the Hall coefficient $R_H = -\frac{V_H}{IB} = \frac{1}{qn}$

Remarks about R_H (or V_H):

- It does not depend on the geometry of the sample
- It is a direct way to measure the charge carrier density
- It gives the sign of the charge carrier (test of the ambipolar behavior of graphene or Tis)

2. Hall effect in Graphene



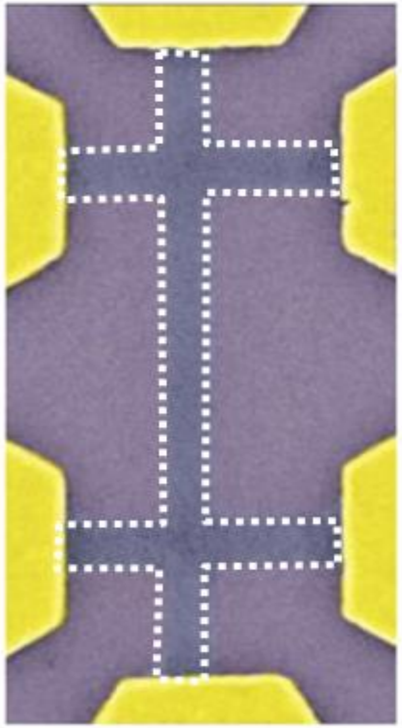


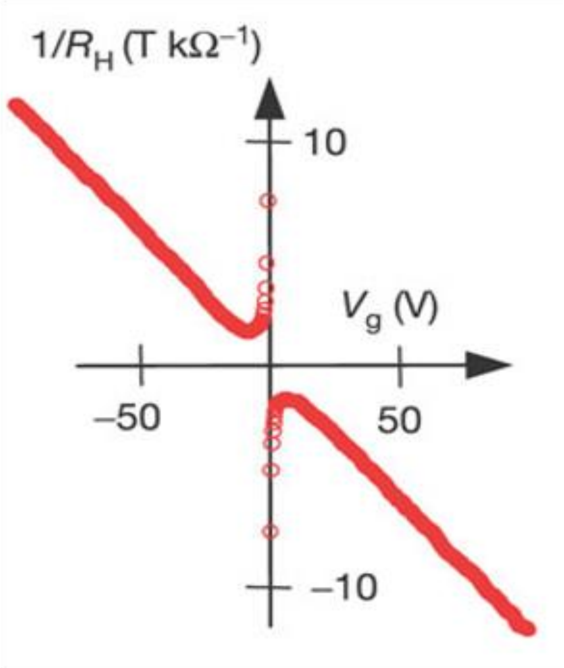
For a given magnetic field, the Hall voltage is given by $V_H = -\frac{BI}{qn(V_G)}$

$\rightarrow$ depends on the gate voltage through the charge carrier density $n(V_G)$.

Judicious to plot the charge density $n(V_G) = -\frac{BI}{qV_H}$ vs. V_G

$\rightarrow$ A change in the sign of $n(V_G)$ (or of V_H) is a clear indication that q changed its sign (e <-> h)



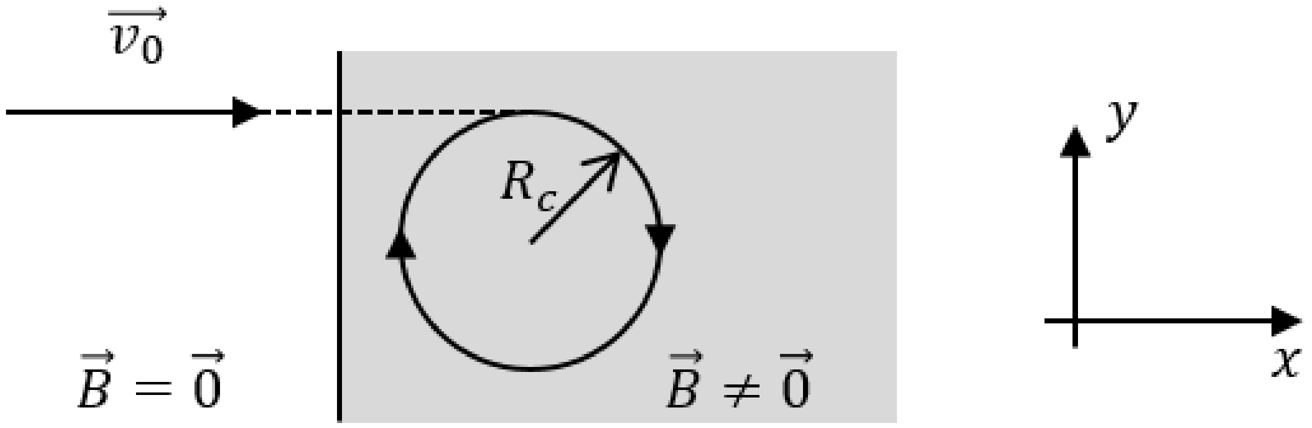


Hall bar patterned in a graphene sheet and measurement of Hall effect that shows a ambipolar behavior (from Novoselov et al.)

3. Case of an infinite 2D-metal with no disorder (massive fermions)

→ Classical derivation

Cyclotron orbitals of the charge (no disorder).



$$\vec{F} = m\frac{d\vec{v}}{dt} = q\vec{v} \wedge \vec{B} \qquad \text{with } \vec{v} = \begin{pmatrix} v_x \\ v_y \end{pmatrix}$$

$$\begin{cases} m\dfrac{dv_x}{dt} = qv_yB \\ m\dfrac{dv_y}{dt} = qv_xB \end{cases}$$

$$\Rightarrow \frac{d^2v_x}{dt^2} + \omega_c^2 v_x = 0$$

where $\omega_c = \frac{qB}{m}$ is the cyclotron frequency

The charge motion describes orbitals so that in the case of a non-disordered material, the switching of the magnetic field leads to a transition to an insulating state (I=0)!

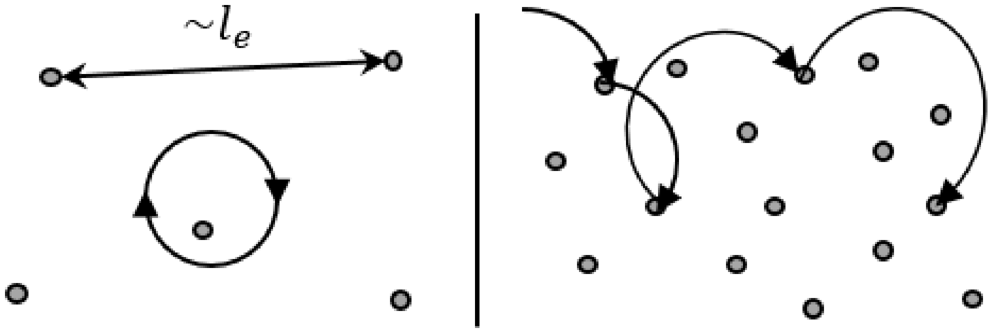
The radius of the orbitals is given by the cyclotron radius:

$$\overrightarrow{v_e} = \begin{pmatrix} v_0 \cos(\omega_c t) \\ v_0 \sin(\omega_c t) \end{pmatrix}$$

$$\begin{pmatrix} X \\ Y \end{pmatrix} = \begin{pmatrix} -v_0/\omega_c \sin(\omega_c t) + x_0 \\ v_0/\omega_c\,(\cos(\omega_c t) - 1) + y_0 \end{pmatrix}$$

$$R_c = \frac{v_0}{\omega_c} = \frac{v_0 m}{qB}$$

We distinguish a clean metal from a disordered one by the ability for charge carriers under magnetic field to form orbitals without to be scattered. A clean metal corresponds to the limit $R_c < l_e$. A non-intuitive consequence of this model is that the introduction of disorder restores the conductance!



Case of a clean metal: $R_c < l_e$ Case of a disordered metal: $R_c > l_e$

Two ways to reach this regime : to improve the material properties (longer l_e) or to use high magnetic field (up to 80T!)…