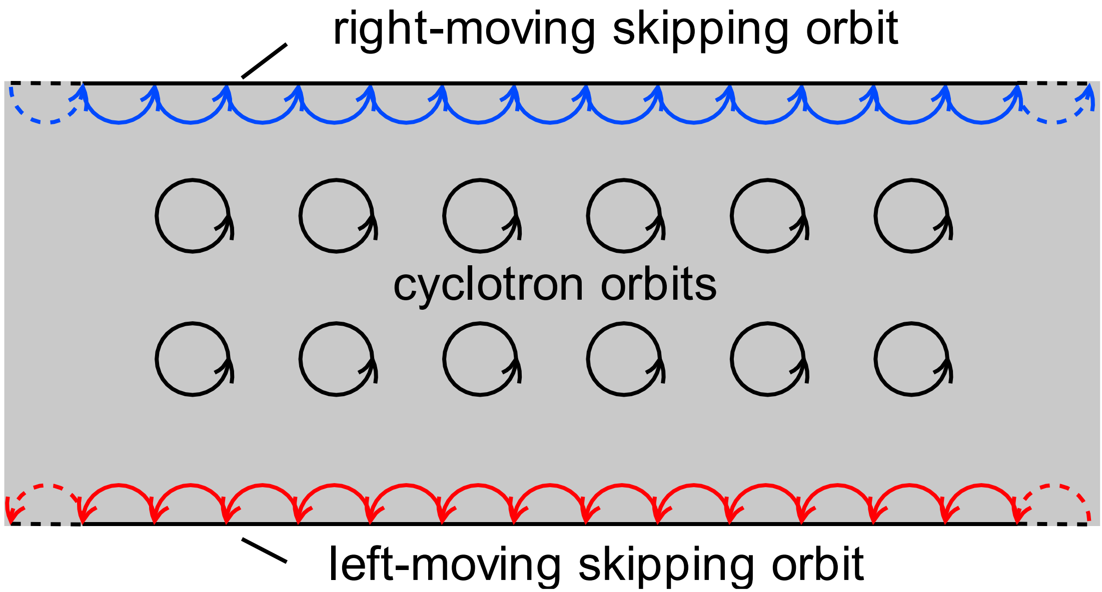
ii. Quantum mechanics

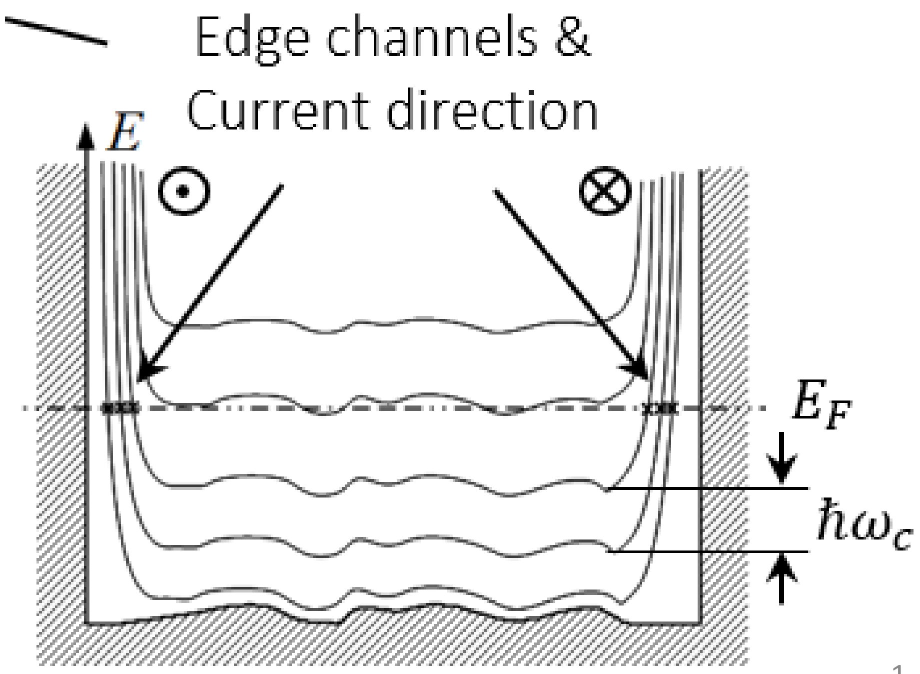


Edges in the sample → infinite repulsive potential outside of the sample in the Hamiltonian

If adiabatic potential → shift the position of the eigen energies.

Good assumption if the potential changes slowly at the scale of R_c.

$$\mathcal{H} = \frac{(\hbar\vec{k} + e\vec{A})^2}{2m} + U_{wall}(x) \;\Rightarrow\; E_n(x) = \hbar\omega_c\left(\frac{1}{2} + n\right) + U_{wall}(x)$$



- Bulk → no influence of the edges. Formation of LLs
- Close to the edges, LLs are shifted up such that **even if the Fermi energy $\boldsymbol{E_F}$ lies in a gap** in the bulk of the sample, **it crosses a LL somewhere close to the edge**.

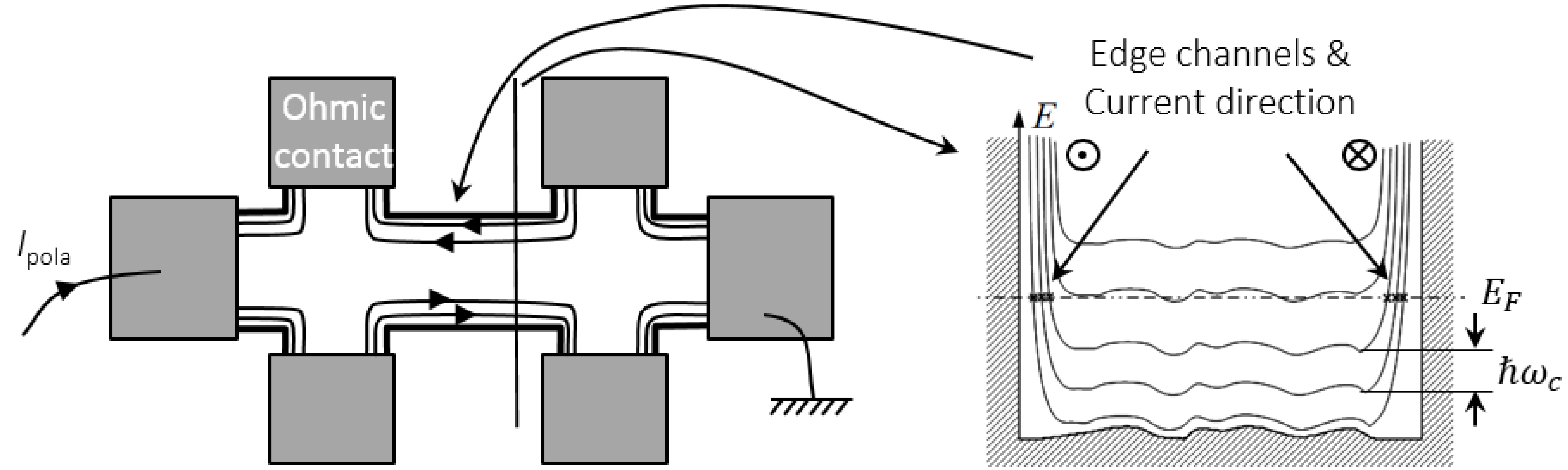
Close to the edges: LLs are not localized states anymore
→ Chiral conduction (classical model).

Edge states / Edge channels:

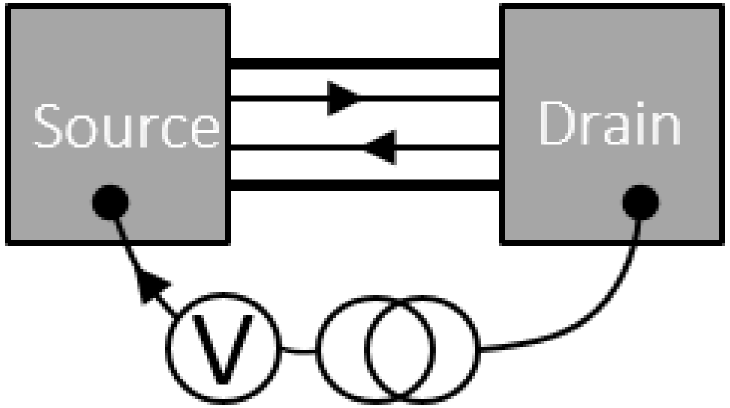
- **purely ballistic (ideal conductors)**
- **chiral**
- **purely 1D channels**

Remarks:

1. number of edge channels = number of LLs below E_f (filling factor ν)
2. magnetic field ↗, number of edge channels ↘.
3. When E_f lies between two LLs, current carried by edge channels only.



6. Conductance of an edge channel



determination of G in a two point configuration of an edge channel

↔ determination of G for a 1D conductor with transmission $T = 1$ (ballistic motion).

i. *First derivation of the conductance of a 1D channel*

(reasonable) <u>assumptions</u>:

1. Each contact emits charges with an emission rate that is determined by its electrochemical potential. The Pauli exclusion and the Heisenberg principle allow to estimate the emission rate: $\Gamma = 2 \times \frac{eV}{h}$
2. Each contact absorbs all incoming charge carrier (black body model)
3. Energy relaxations occurs into the contact only (length of the channel $< l_{in}$)

If ballistic ($T = 1$), flux of charges Source → Drain:

$$N_{S \to D} = \frac{2eV_S}{h}$$

Similarly, flux of charges Drain → Source:

$$N_{D \to S} = \frac{2eV_D}{h}$$

total current is obtained by doing the difference of both quantities:

$$I = e(N_{S \to D} - N_{D \to S})$$

$$I = \frac{2e^2}{h}(V_S - V_D)$$

Conductance of a 1D ballistic conductor

Important remarks:

- the conductance **is not infinite** even if for **a perfectly** transmitted channel
- the conductance is related to the **fundamental constants e** (the charge of an electron) and **h** (the Plank constant). It does not depend on the geometry of the sample nor on the charge density!

ii. Semi-classical derivation of the conductance of a 1D channel

In this demonstration, we need the same assumptions as previously.

The current is given (at zero temperature) by:

$$I = \frac{1}{2}\int_{E_c}^{eV_S} e \times n_{1D}(E) \times v(E)dE - \frac{1}{2}\int_{E_c}^{eV_D} e \times n_{1D}(E) \times v(E)dE$$

The factor ½ comes from the fact that half of the charge carriers have a positive velocity, the rest having a negative velocity.

$$n_{1D}(E) = \frac{dN_{1D}(E)}{dE} = \frac{d}{dE}\int_{-k(E)}^{k(E)} n_k dk = \frac{2s}{2\pi}\frac{dk}{dE}$$

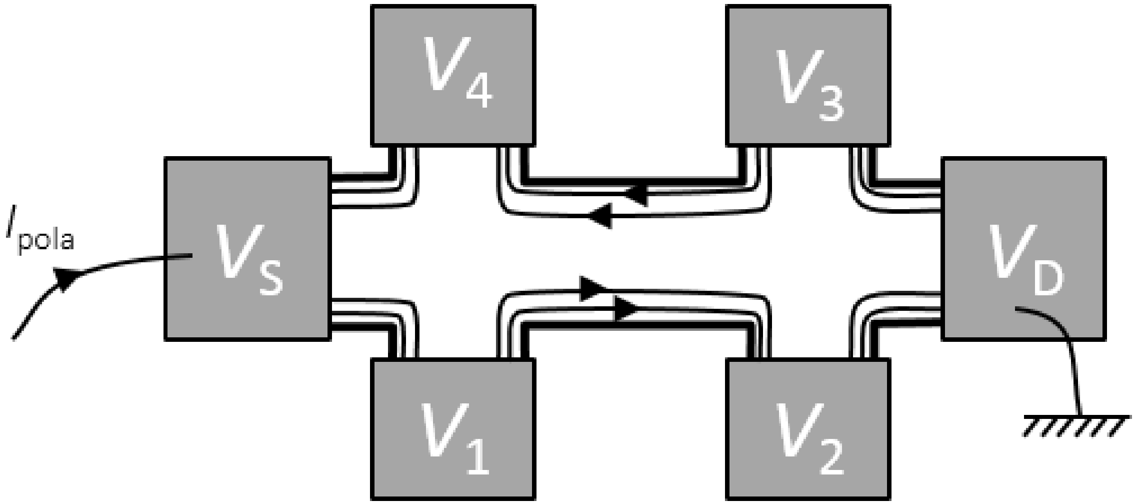
With s the spin degeneracy. Taking into account the fact that $v(E) = \frac{1}{\hbar}\frac{dE}{dk}$ we obtain finally:

$$n_{1D}(E) = \frac{2s}{hv(E)}$$

Which results in $I = \int_{eV_D}^{eV_S} e \times \frac{s}{h} dE = s\frac{e^2}{h}(V_S - V_D)$

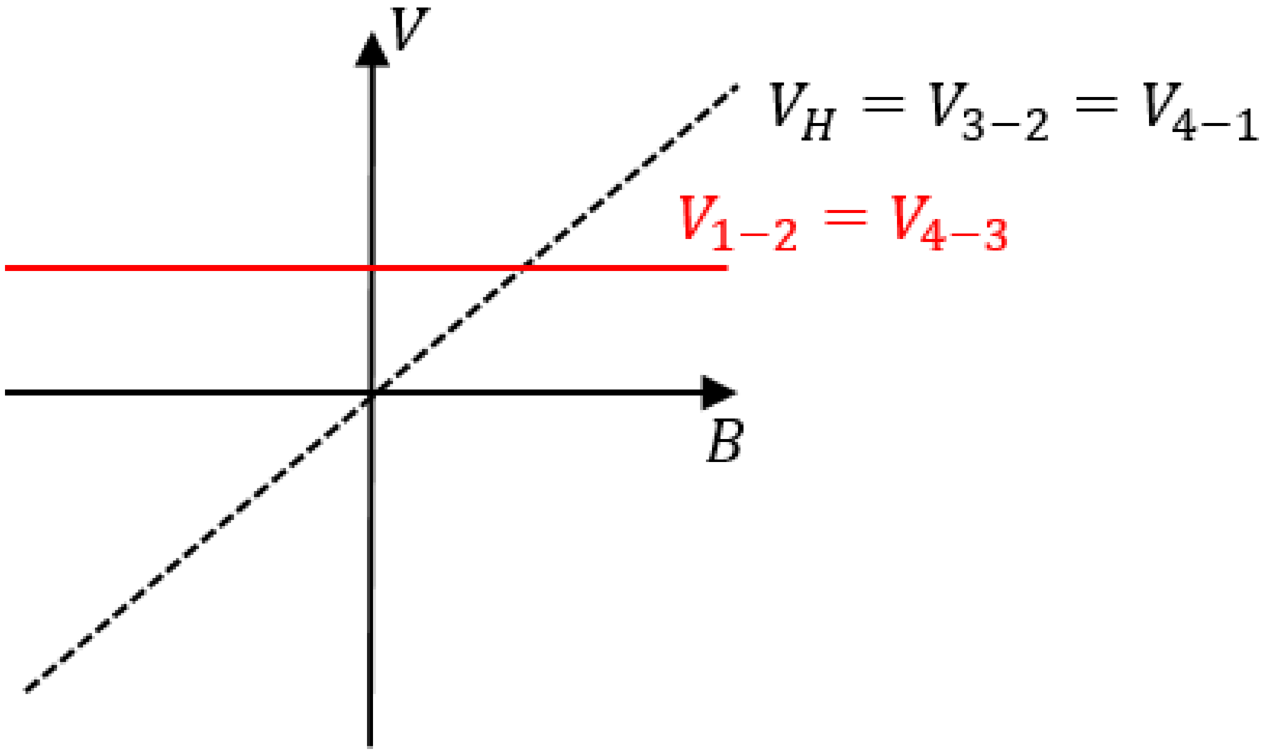
This relation is very general and does not depend on the dispersion relation.

7. Hall bar in the Quantum Hall effect regime



Hall voltage and longitudinal voltage measured on a Hall bar in the different field regimes:

- hall regime
- Shubnikov-de Haas regime
- QHE regime.



i. The classical Hall regime

This regime corresponds to $\mu B = \omega_c \tau = R_c / l_e \lesssim 1$

7. Hall bar in the Quantum Hall effect regime

Hall voltage and longitudinal voltage measured on a Hall bar in the different field regimes:

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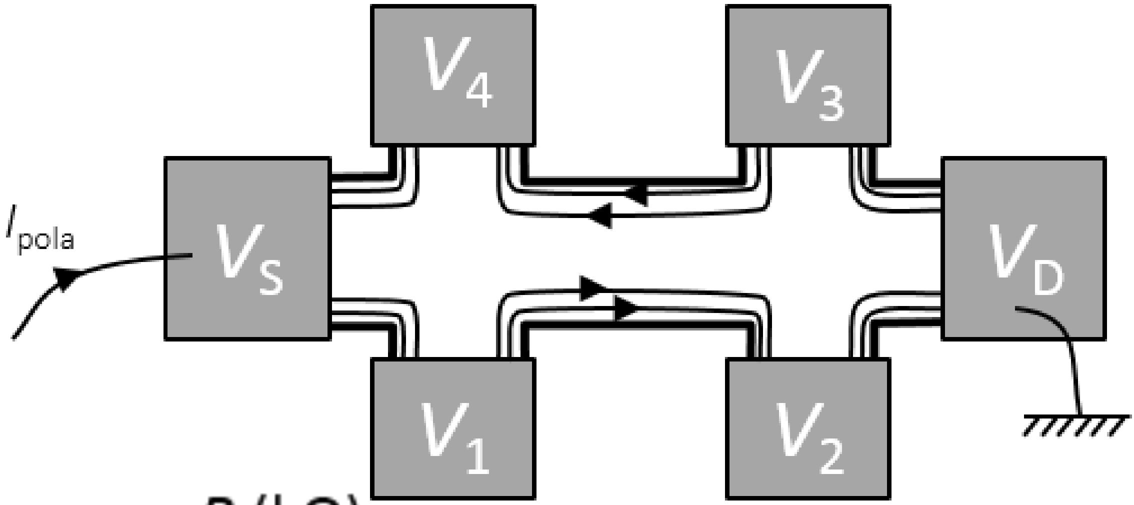
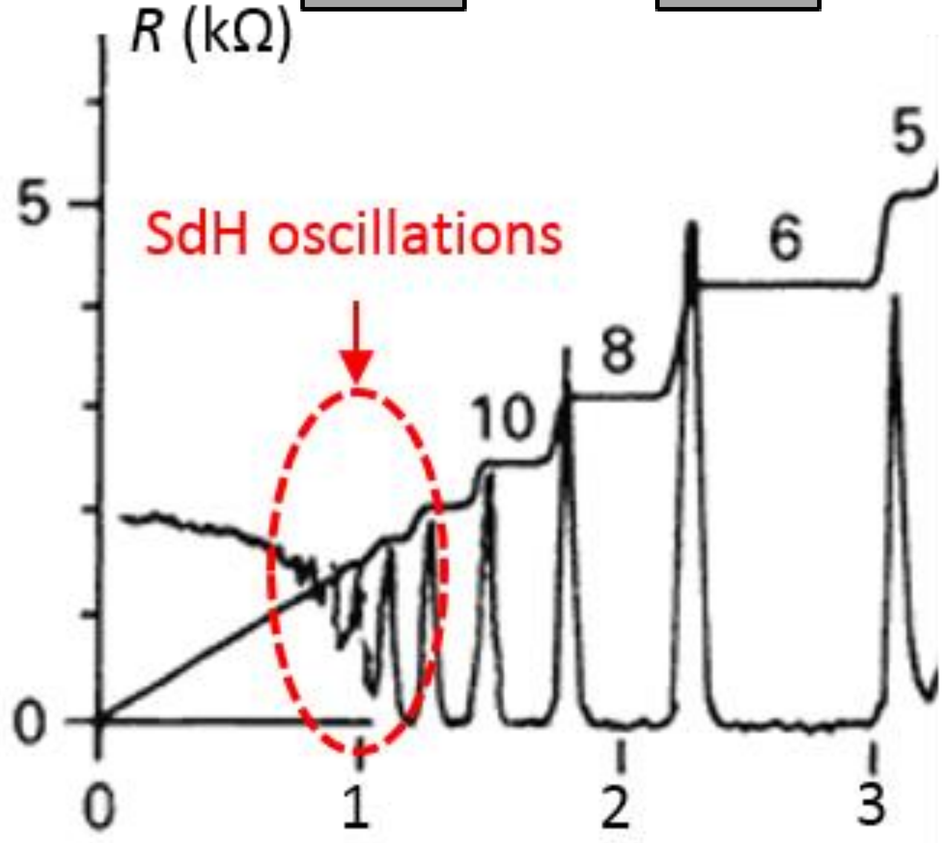




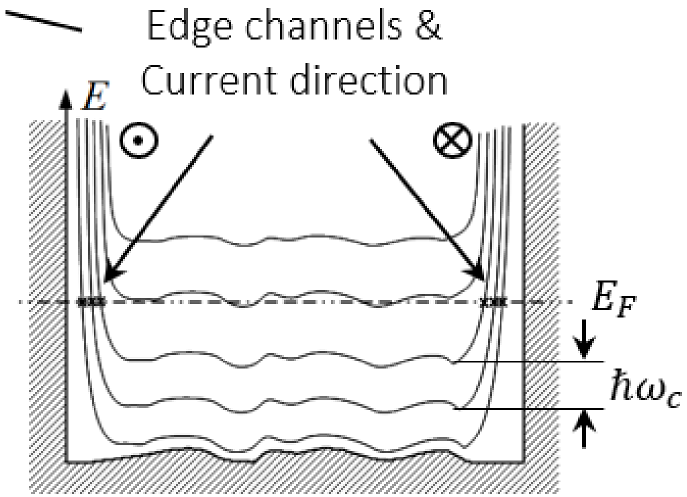
ii. The Shubnikov-de Haas regime

This regime corresponds to $\mu B = \omega_c \tau = R_c / l_e \gtrsim 1$

and $\lambda_F \ll l_B = \sqrt{\frac{h}{eB}}$ where $l_B$is the magnetic length

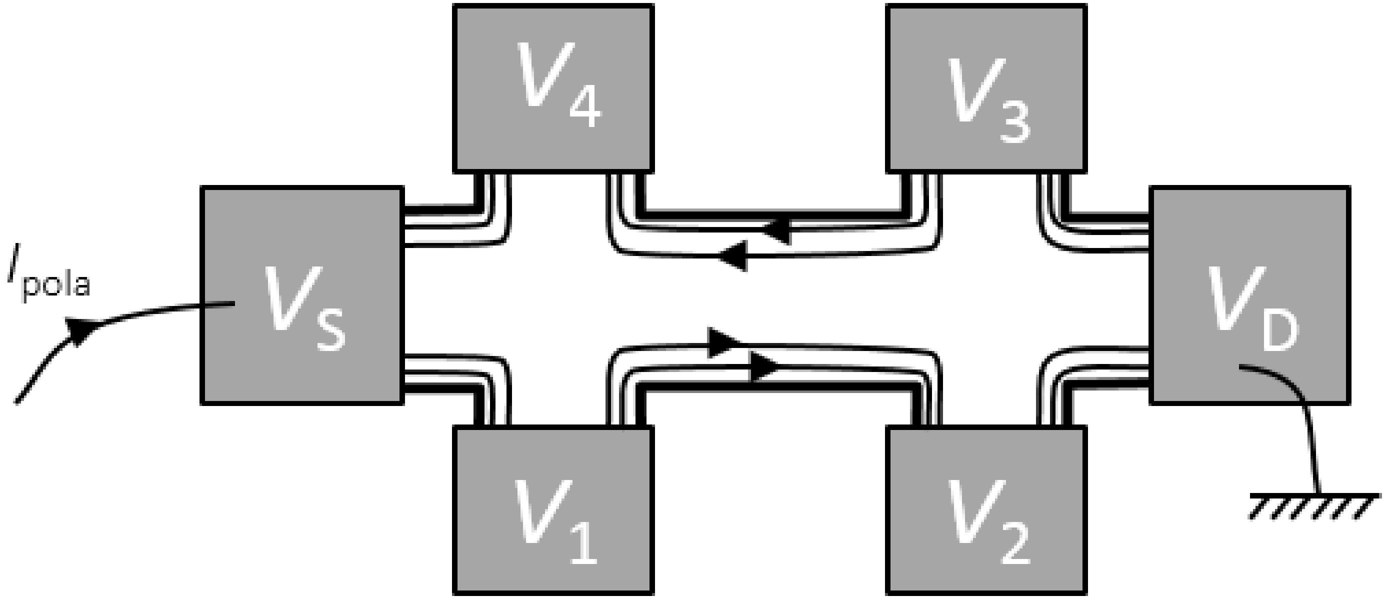
iii. The quantum Hall regime

This regime corresponds to $\lambda_F \sim l_B$. In the practice, we consider that the QHE is reached **once the longitudinal resistance completely vanishes** and that the Hall resistance simultaneously shows a **well-defined plateau**. We consider the case of



$$\upsilon = 1 \Leftrightarrow E_F < \frac{3}{2}\hbar\omega_c \Leftrightarrow B > \frac{2}{3}E_F\frac{m}{e\hbar}$$

In the quantum Hall regime, the **right moving charge carriers are physically separated from the left moving charge carriers** (chirality of the edge channels). We can consider the different contact one by one:

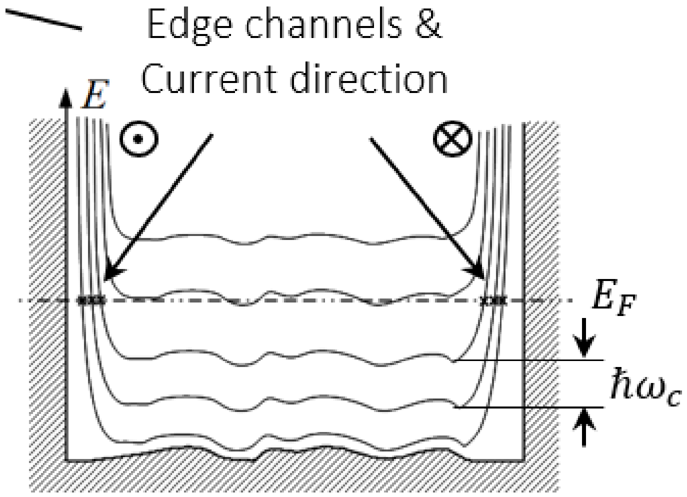


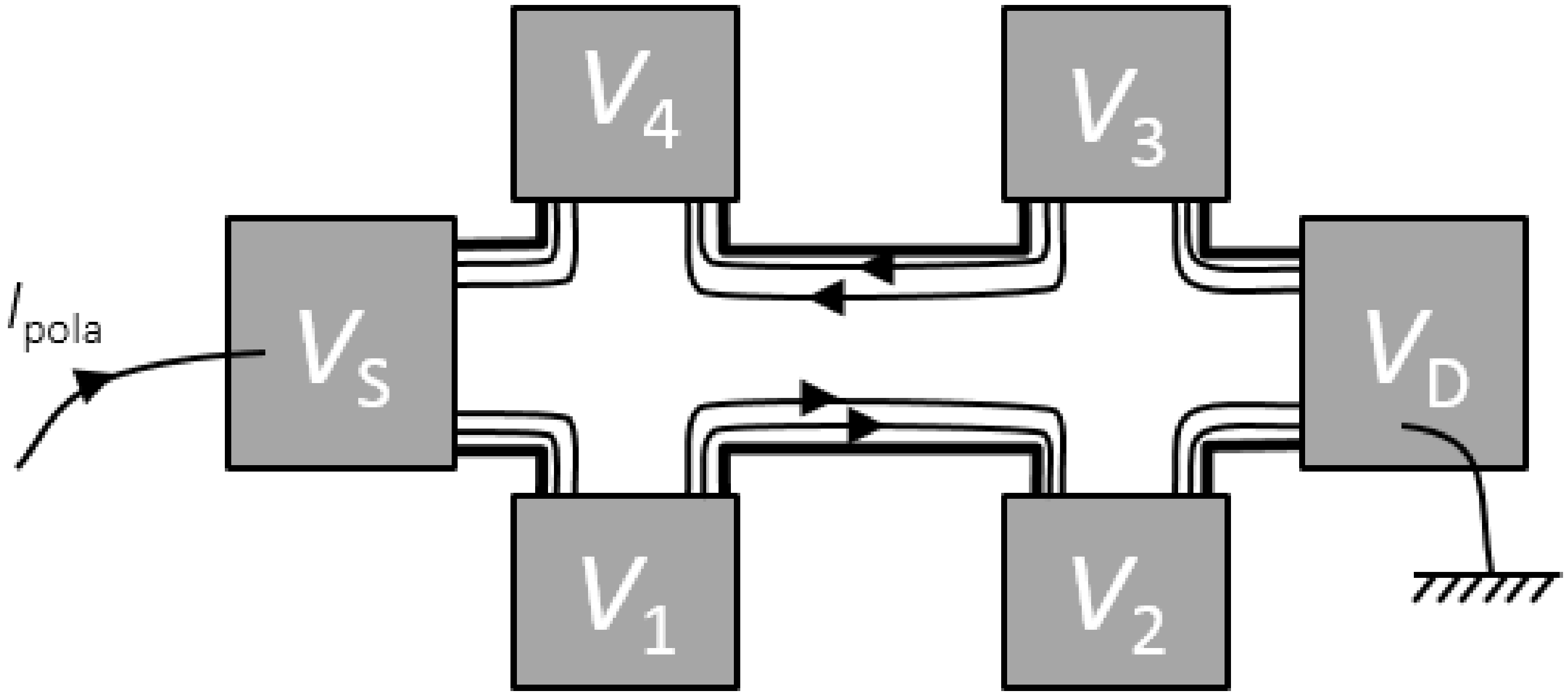
Source contact

- absorbs (from 4): $\frac{2eV_4}{h} = N_{\to S}$ (ballistic)
- emits (to contact 1): $\frac{2eV_S}{h} = N_{S\to}$

$$\Rightarrow I_S = I = e(N_{S\to} - N_{\to S})$$

$$I = 2\frac{e^2}{h}(V_S - V_4)$$



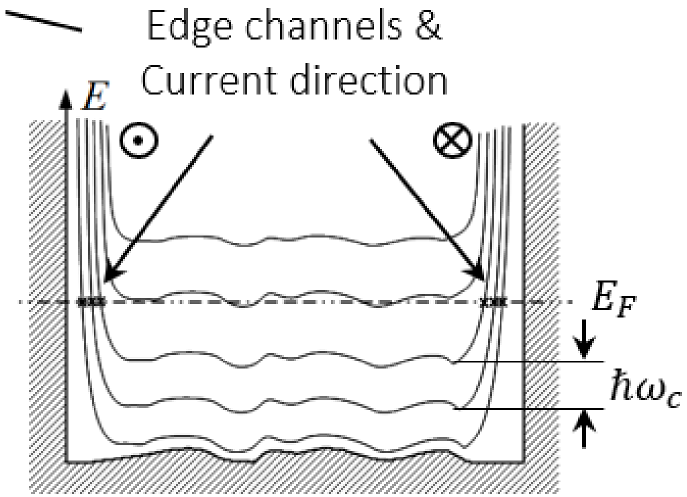


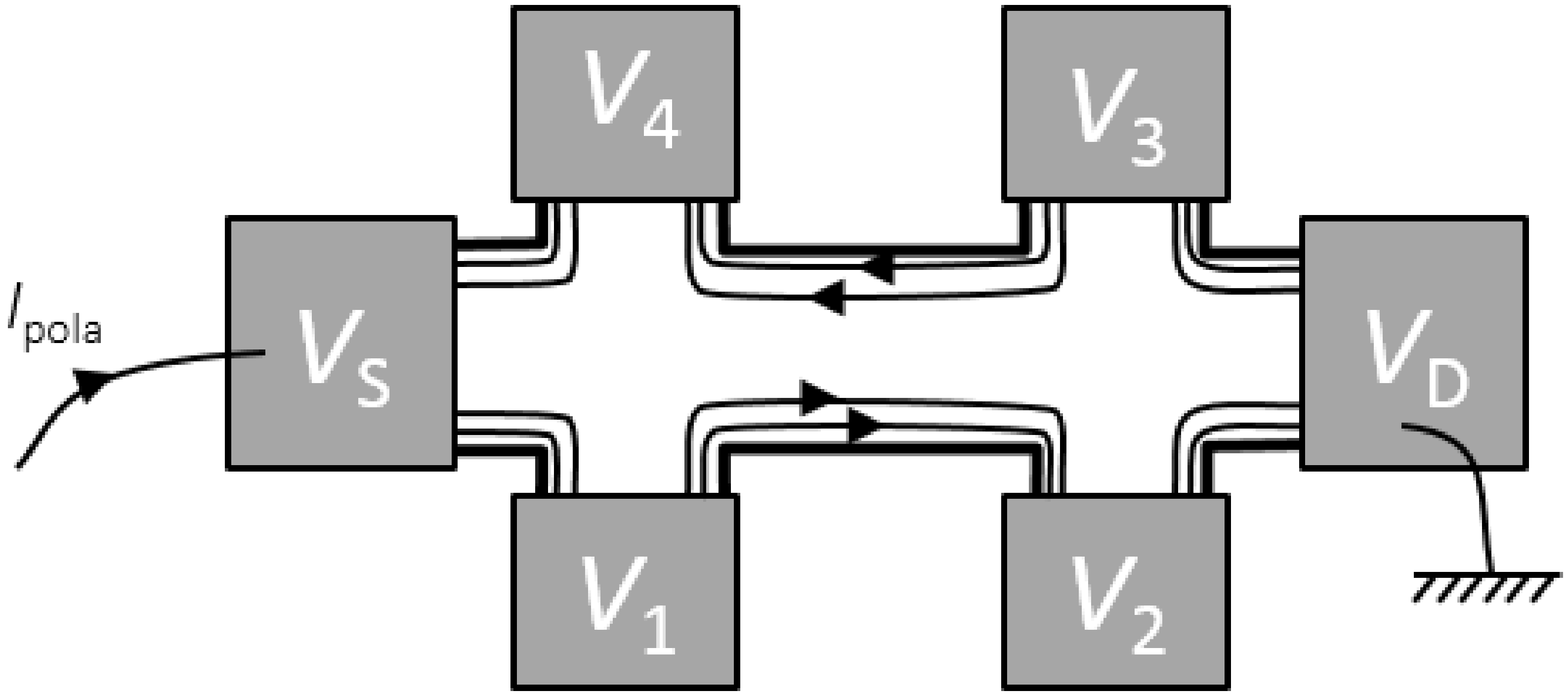
Contact 1

- absorbs (from the Source): $\frac{2eV_S}{h} = N_{\to 1}$
- emits (to contact 2): $\frac{2eV_1}{h} = N_{1\to}$

$$\Rightarrow I_1 = 0 = e(N_{\to 1} - N_{1\to})$$

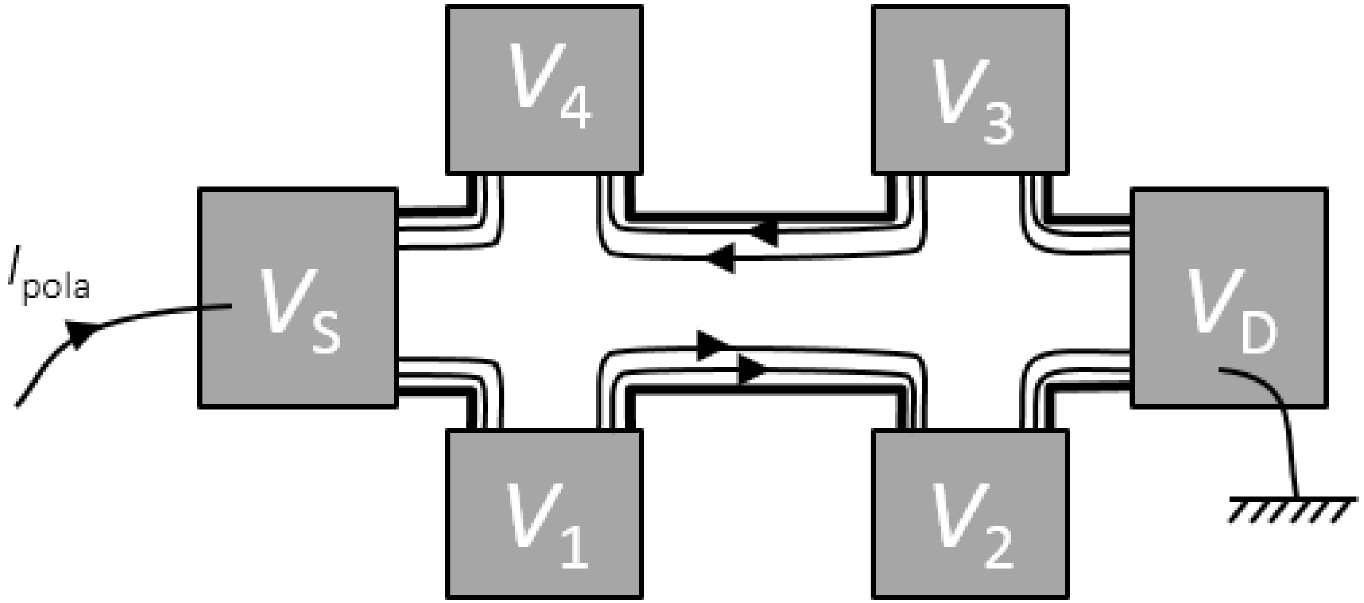
$$\Rightarrow V_1 = V_S$$





The same reasoning leads to

$$I_2 = 0 \Rightarrow V_2 = V_1$$
$$I_D = -I = 2\,e^2/h\,(V_D - V_2)$$
$$I_3 = 0 \Rightarrow V_3 = V_D$$
$$I_4 = 0 \Rightarrow V_4 = V_3$$



$$\Rightarrow \begin{cases} R_{XX} = (V_1 - V_2)/I = (V_3 - V_4)/I = 0 \\ I/V_{Hall} = I/(V_1 - V_4) = I/(V_2 - V_3) = 2 \times \dfrac{e^2}{h} \end{cases}$$

$$\boxed{\Rightarrow R_{XX} = 0 \quad \text{and} \quad R_{Hall} = 2 \times \frac{e^2}{h}}$$

$$\Rightarrow R_{XX} = 0 \quad \text{and} \quad R_{Hall} = 2 \times \frac{e^2}{h}$$

At $\upsilon = 1$, V_{Hall} exhibits a plateau which value is given by **fundamental physics constant** (e and h).

It does not depend on the geometry nor on any other parameters of the metal (density, mean free path, effective mass,…).

Hall resistance of the plateau = Klitzing constant $R_K = \left(\frac{e^2}{h}\right)^{-1} = 25\,812, \ldots \Omega$

→ definition of the quantum of conductance/resistance.

Robustness: QHE used in metrology to determine the value of R_K.

At $\upsilon = n$, the emission rate of charge carrier of a contact is given by $\frac{2eV}{h}$ ***<u>per channel</u>***:

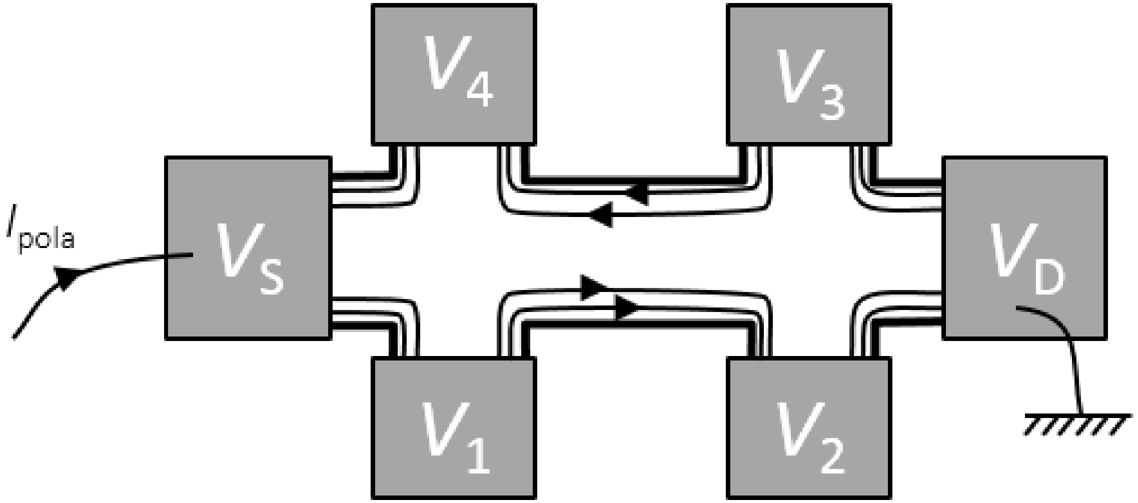
$$\Downarrow$$

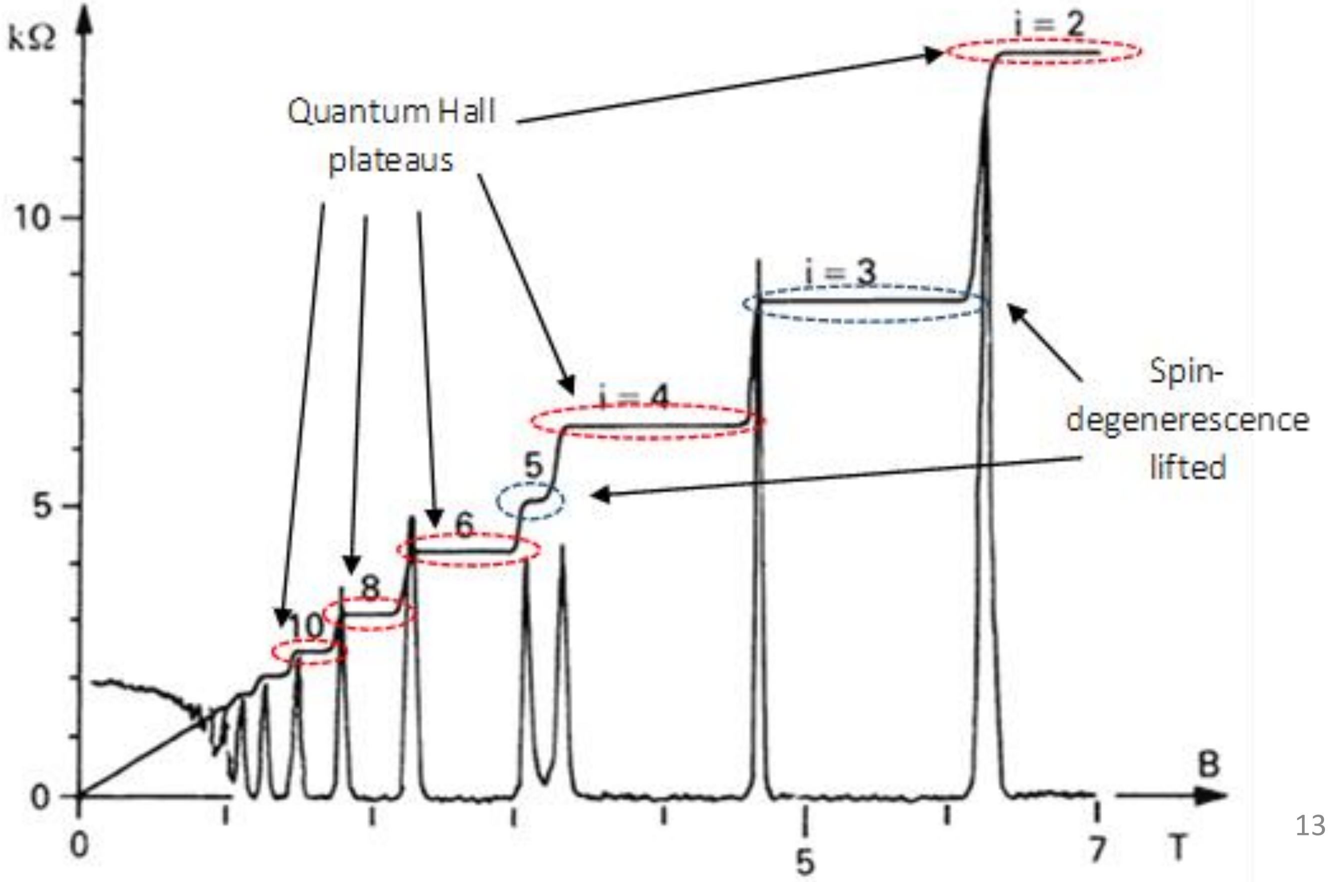
$$V_H = \frac{1}{n} \times \frac{2e^2}{h} I$$

in a four probe configuration: longitudinal resistance = 0

(in agreement with the common sense) ≠ two probe measurements.

<u>resistance of the 1D perfectly transmitted channel in a two probe</u>: access resistance due to the strong reduction of the number of conducting channels between the macroscopic contact and the 1D conductor





8. Landau Levels and the quantum Hall effect in Graphene

Hall bar of graphene & longitudinal resistance at higher magnetic field ($\mu B \gtrsim 1$).

The QHE regime is expected to be very different than the one of massive fermions since $m \to 0$.

Massive fermions, the cyclotron frequency is given by $\omega_c = \frac{eB}{m} \to \infty$ when $m \to 0$!

The energy of the LLs should diverge:

$$E_n = \hbar\omega_c\left(\frac{1}{2} + n\right) \xrightarrow[m \to 0]{} \infty$$

Breakdown of the massive model in the case of massless Dirac fermions…

It can be shown than in the relativistic case, the cyclotron frequency can be given by $\omega_c = \sqrt{2}\frac{v_F}{l_B}$ and the cyclotron radius

has to be written in a more general way as $R_c = \frac{v_F}{\omega_c}$ (does not depend on the mass any more).

Similar argument than previously (semi-classical approach): $\boldsymbol{k}$-vector is quantized along cyclotron orbital

$$\Leftrightarrow k = N/R_c \text{ with } N \in \mathbb{N}$$

and dispersion relation at zero magnetic field : $E = \pm\hbar v_F|k|$ so that

$$E_N = \pm\hbar v_F \frac{N}{R_c} = \pm\hbar\omega_c N \text{ (Wrong formulae!)}$$

As in the case of massive fermions, wrong formulae but gives the key ingredients of the quantum Hall effect with massless fermions and particularly the fact that

$$E_N \propto \sqrt{B}$$

Whereas $E_N \propto B$ for massive fermions.

Solving the Hamiltonian of massless Dirac fermions under a magnetic field gives the following eigenvalues E_N:

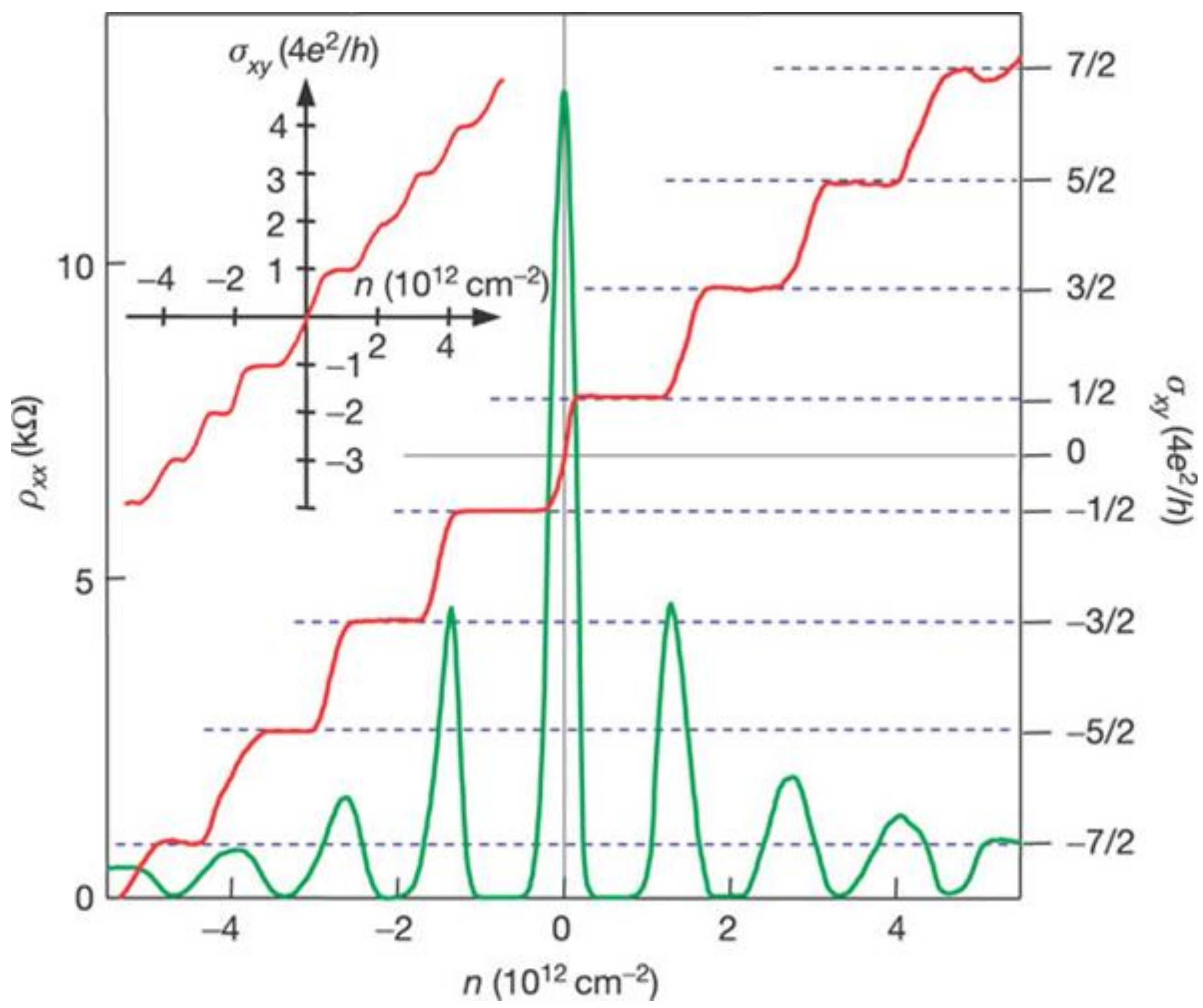
$$E_N = \pm\hbar\omega_c\sqrt{N}$$

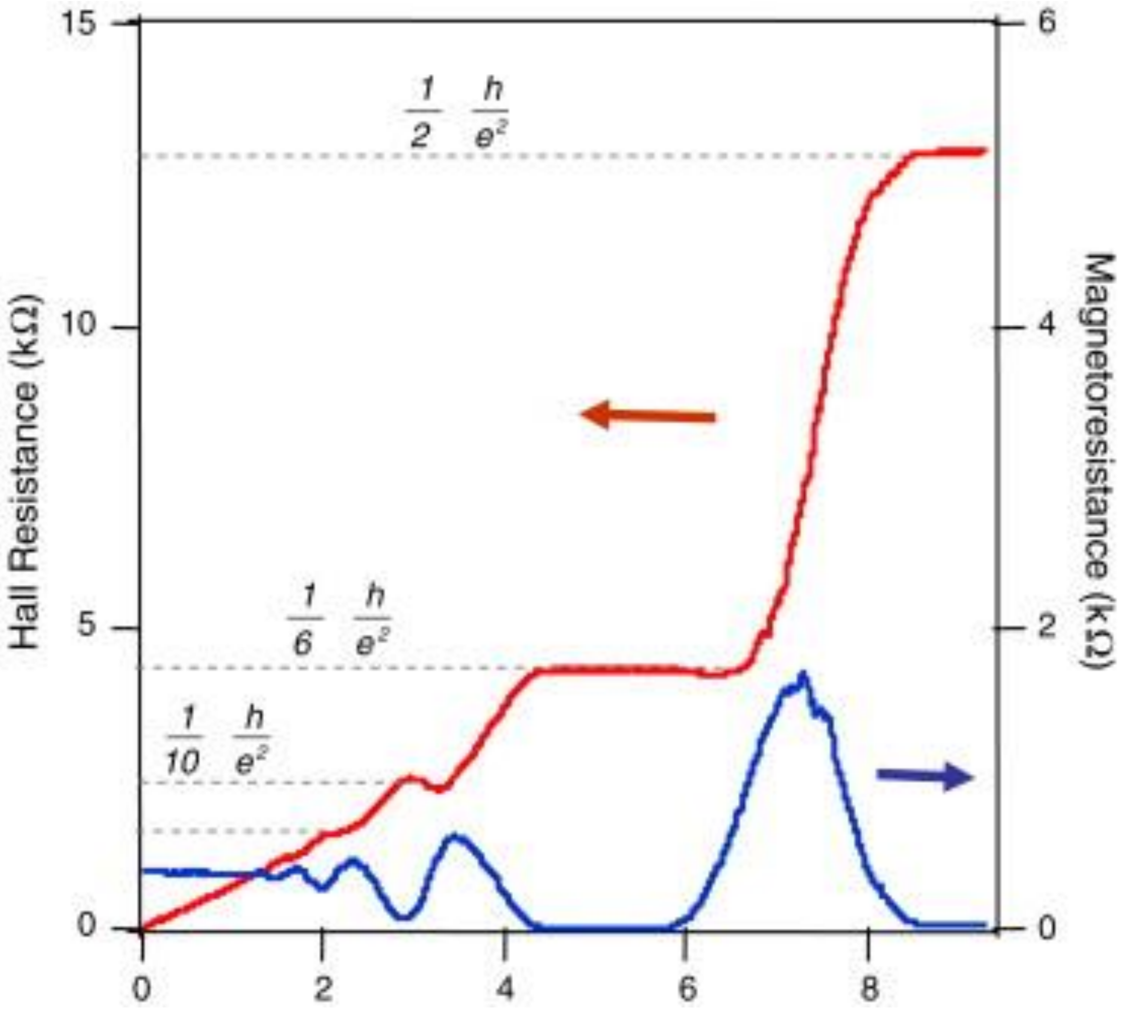
Very important differences with the case of massive particles have to be mentioned:

- The LLs are not equally placed in energy
- There is a zero energy mode
- There is positives (electrons) as well as negatives (holes) solutions

Even if the cyclotron energy in graphene does not $\rightarrow \infty$, it remains much larger than the one of usual massive particles. Typically, $\hbar\omega_c{\sim}1000K$ at $B = 10T$. This make possible the observation of the quantum Hall effect at room temperature!

Example of the first experimental observation of such quantum Hall effect for massless Dirac Fermions (from Novoselo *et al.*):





≠ case of the QHE in GaAs based heterostructures (massive):

the LLs have just a spin degeneracy

→ plateaus at multiple of $2 \times e^2/h$ instead of e^2/h

in the case of graphene: spin degeneracy **+** valley degeneracy

(two valleys K and K' in the Brillouin zone)

→ steps of $4 \times e^2/h$.

!!!: The zero energy LL of zero energy is **equally composed of electrons and holes**. → Degeneracy of 2 instead of 4.

In a similar picture than the one we used for electrons (LLs shifted at high energy when approching the edges of the sample):

the LLs of holes shifted at low energy. Zero energy LL **is splitted into two LLs**: e- and h+. Each of those LLs is two times lower.
→ plateaus of $2\,e^2/h$ (instead of $4\,e^2/h$).

The symmetry electron-hole is respected.