

Dephasing caused by the Thin Spectrum in a BCS Superconductor

Jasper van Wezel and Jeroen van den Brink

Institute-Lorentz for Theoretical Physics, Universiteit Leiden, P.O. Box 9506, 2300 RA Leiden, The Netherlands

Abstract. Recently we have shown that the thin spectrum associated with the process of spontaneous symmetry breaking will lead to a maximum coherence time for many-particle solid state qubits [1, 2, 3]. In this paper we will discuss how the general mechanism of dephasing caused by a thin spectrum can be applied to the case of BCS superconductors. We will find that the BCS superconducting state is a state of broken global U(1) symmetry, and that the thin spectrum associated with the breaking of this phase symmetry induces dephasing and decoherence within the fundamental timescale $\tau = 2\pi N\hbar/k_B T$.

Keywords: Superconductivity, Spontaneous Symmetry Breaking, Decoherence

PACS: 03.65.Yz, 11.30.Qc, 74.20.Fg

INTRODUCTION

Recently we have shown that spontaneous symmetry breaking imposes a fundamental limit for the time that a many-particle solid-state qubit can stay quantum coherent. This universal timescale is $\tau = 2\pi N\hbar/k_B T$, where N is the number of constituent particles involved in the superposed state [1]. We have shown this timescale to be applicable to qubit systems based on crystals, antiferromagnets and even superconductors [2, 3]. In the case of superconducting qubits we have considered a local pairing, negative- U Hubbard model to describe the superconducting state [3]. The goal of this paper is to show that the result on decoherence in a superconductor does not depend on the specific model used. To do so, we will consider the standard BCS model for superconductivity and show that it too has a thin spectrum which causes the dephasing of the computational states of a superconducting qubit and thus gives rise to the universal maximum coherence time τ .

The many-particle qubits that motivate us to study decoherence due to spontaneous symmetry breaking in superconductors are realized in a number of mesoscopic solid state systems. For instance, by engineering aluminum on a sub-micron length scale, superconducting Cooper pair boxes can be manufactured. A Cooper pair box is a superconducting island, containing $N \sim 10^8$ electrons, which can be brought into a superposition of two states with a different number of Cooper pairs [4, 5].

THE BCS SUPERCONDUCTOR

Previously we have shown that the presence of a thin spectrum leads to dephasing and decoherence in a local pairing model for superconductivity [3]. It could be argued that such a local pairing model is somewhat pathological, and not really representative of

CP918, *Lectures on the Physics of Strongly Correlated Systems XI:
Eleventh Training Course*, edited by A. Avella and F. Mancini
© 2007 American Institute of Physics 978-0-7354-0428-1/07/\$23.00

real-life superconductors, even though from the point of view of symmetry the model is equivalent to a weak coupling model (because there is no phase transition which separates the two [6]). In this paper we will therefore study the symmetry breaking and dephasing in a BCS description, and show that although the picture changes slightly, the underlying physics is exactly equivalent, and in fact gives rise to the exact same conclusions regarding the thin spectrum and the timescale on which decoherence will set in. The draw-back of doing the calculation in the BCS description is that we cannot do it in a manifestly gauge invariant way, so that the role of the vector potential is obscured.

After creating Cooper pairs, we arrive in the standard BCS theory at the effective Hamiltonian [7]

$$H_{BCS} = \sum_k \varepsilon_k \left(c_k^\dagger c_k + c_{-k}^\dagger c_{-k} \right) - \sum_{k \neq k'} U_k c_k^\dagger c_{-k}^\dagger c_{-k'} c_{k'}.$$

Here we have adopted the convention to write (k, \uparrow) as k and $(-k, \downarrow)$ as $-k$. The dispersion of the bare Fermi-sea is characterized by ε_k while U_k is the effective pairing interaction due to phonon exchange. U_k is non-zero and attractive only in a shell around the Fermi energy with a width of about the Debye energy. It is easy to see that extensivity of the model in fact requires U_k to be inversely proportional to the total number of electrons in the system. We will therefore redefine the pairing potential as $U_k = V_k/N$, where N denotes the total number of electrons in the k -space shell in which U_k is non-zero.

By writing down the Hamiltonian (1) we have assumed that there is no external magnetic field and we have fixed the gauge to ensure that the electromagnetic vector potential vanishes everywhere (to include the full vector potential, we would have to let the phase of the hopping parameter depend on the gauge field, and change the Hamiltonian accordingly). Anderson showed that the BCS Hamiltonian in this form can be rewritten as a spin problem by introducing the pseudospins [7]

$$S_k^+ = c_{-k} c_k \quad ; \quad S_k^z = \frac{1}{2} \left[1 - c_k^\dagger c_k - c_{-k}^\dagger c_{-k} \right]. \quad (1)$$

In the subspace without any quasiparticles (i.e. $n_k = n_{-k} \forall k$), the Hamiltonian up to an overall constant becomes

$$H_{BCS} = -2 \sum_k \varepsilon_k S_k^z - \sum_{k \neq k'} \frac{V_k}{N} (S_k^x S_{k'}^x + S_k^y S_{k'}^y). \quad (2)$$

Interpreted at face value, this Hamiltonian describes pseudo spins on a lattice which has position-label k . On this lattice, three different and independent regions can be identified. In the region $k < k_F - k_D$ (where k_F is the Fermi wavenumber and k_D the Debye wavenumber) we know that the pairing potential vanishes and ε_k is negative, so that all pseudospins in that region will point down, which corresponds to completely filled electronic states. In the region $k > k_F + k_D$ the pairing potential is zero as well, but here ε_k will be positive, causing all spins to point up, and all electronic states to be empty. In the shell of width k_D around k_F a more interesting situation occurs. There V_k is nonzero (and approximately constant), while ε_k switches sign right at k_F . The pseudo-spin structure that one would classically expect in that region is that of a magnetic

domain wall: the pseudospins point up at one end of the region, then continuously fall over until they reach the xy plane exactly at k_F , and then they continue on until they point down at the other end [7]. Electronically that structure corresponds to the BCS wavefunction $\prod_k \left(u_k + v_k c_k^\dagger c_{-k}^\dagger \right) | \text{vac} \rangle$ [8].

The Hamiltonian H_{BCS} however is invariant under rotations around the z -axis, and thus the groundstate will also obey this symmetry and have a completely delocalized projection of the pseudospins on the xy plane. To form a true domain wall, and thus the classical superconducting state, this $U(1)$ symmetry will have to be spontaneously broken.

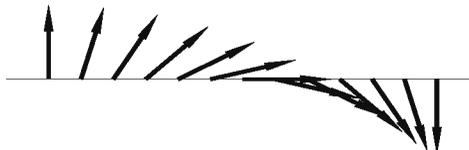


FIGURE 1. A schematic representation of the region of width k_D around k_F . The vectors represent the pseudospins \vec{S} . Spontaneous symmetry breaking causes the projections of the pseudospins in the horizontal plane perpendicular to the paper to align.

Because the symmetry breaking will only have an effect in the region around k_F and because this region is fully decoupled from the other two regions of k -space, let's focus solely on that shell from now on, and define all sums over k to run from $k_F - k_D$ to $k_F + k_D$. In this region the pairing potential is approximately constant and thus we set $V_k \equiv V$ there. The collective dynamics of the system which govern the process of spontaneous symmetry breaking will in fact only be described by the singular points of the Bogoliubov transformation which diagonalizes the Hamiltonian [2]. Because of the ferromagnetic sign, the collective model in this case consists of only the $k = 0$ part of equation (2):

$$H_{coll} \simeq -\frac{V}{N} \left[\vec{S}_{tot} \cdot \vec{S}_{tot} - S_{tot}^z S_{tot}^z \right], \quad (3)$$

where $S_{tot} \equiv \sum_k S_k$ and where we have neglected terms of order $1/N^2$. We have also made a strong coupling approximation by setting $\epsilon_{tot} = 0$. Other approximations of ϵ_{tot} are possible, but it turns out that after some tedious mathematics these will give the exact same form for the thin spectrum and the maximum coherence time as the simple approximation $\epsilon_{tot} = 0$. The eigenstates of the collective Hamiltonian are trivially found to be labeled by the total spin quantum number S and its z -projection M , while the corresponding energies are given by $E_{coll}(S, M) = -V/N (S(S+1) - M^2)$. The thin spectrum in this case is labeled by M , and describes states with different total electron densities. The total spin excitations labeled by S on the other hand, are gapped with an energy $\sim V$. To break the xy -symmetry of H_{coll} we can add a symmetry breaking field $-BS_{tot}^x$ along for example the x -axis. After evaluating its matrix elements [2] and taking the continuum limit, Schrödinger's equation can be written as a harmonic oscillator equation

$$-\frac{1}{2} \frac{\partial^2}{\partial M^2} \Psi(M, x) + \frac{1}{2} \omega^2 M^2 \Psi(M, x) = v \Psi(M, x), \quad (4)$$

with $\omega^2 = (2V)/(BNS)$ and $v = 1 + (E(S,x) - E_{coll}(S,0))/(BS)$. The symmetry broken wavefunctions $|S,x\rangle \equiv \sum_M \Psi(M,x) |S,M\rangle$ thus have energies

$$E(S,x) = -\frac{V}{N}S(S+1) - BS + \left(x + \frac{1}{2}\right) \sqrt{VB} \sqrt{\frac{2S}{N}}. \quad (5)$$

In the ground state S will be maximal (i.e. $N/2$), and then the term $\propto NB$ in the energy signals spontaneous symmetry breaking: in the thermodynamic limit the system can gain an infinite amount of energy by aligning with an infinitesimally small symmetry breaking field [9].

DECOHERENCE

The collective excitations that make up the (dual) thin spectrum on top of the symmetry broken ground state are labeled by x . Their energies are slightly influenced by the remaining collective quantum number S . If we were to use this superconductor as a qubit then we would have to use the states labelled by S as the computational states of the qubit system. After all, the thin spectrum states labelled by x are so sparse that they cannot be seen in experiment [2]. The total spin excitations in this model correspond to changing the total number of cooper pairs in the electronic Hamiltonian (1). The superposition state that we are describing should therefore be closely related to the superposition state that is experimentally realized in Cooper-pair box qubits [4, 5]. If we make a superposition of total spin states and trace away the unobservable thin spectrum, then the small shift in the thin spectrum's energy levels will cause a dephasing of the computational states and subsequently the decoherence of the visible, reduced density matrix, in a manner completely analogous to the introduction of decoherence in a crystal, antiferromagnet or local pairing superconductor [1, 2, 3]:

$$\begin{aligned} \rho_{t=0} &= \frac{1}{2Z} \sum_x e^{-\beta E(x,0)} [|x,0\rangle + |x,1\rangle][\langle x,0| + \langle x,1|] \\ \rho_{t>0} &= \frac{1}{2Z} \sum_x e^{-\beta E(x,0)} [|x,0\rangle \langle x,0| + |x,1\rangle \langle x,1| \\ &\quad + e^{-\frac{i}{\hbar}(E(x,0)-E(x,1))t} |x,0\rangle \langle x,1| + \text{h.c.}], \\ \rho_{t>t_0}^{OD} &= \frac{1}{Z} \sum_x e^{-\beta E(x,0)} e^{-i(E(x,0)-E(x,1))(t-t_0)/\hbar}. \end{aligned} \quad (6)$$

Here Z is the partition function at time $t = 0$, ρ_t is the density matrix at time t , and ρ_t^{OD} is the off diagonal element of the reduced density matrix which is obtained by tracing the full density matrix over the thin spectrum states. The resulting maximum coherence time of the superposition state within the BCS superconductor is given by the half-time of ρ_t^{OD} , which turns out to be

$$\tau = \frac{2\pi\hbar}{k_B T} N, \quad (7)$$

where N counts the number of states in the k space volume of k_D around k_F , which is proportional to the number of Cooper pairs in the superconducting condensate. Notice that we find the same universal form for the expression of the coherence time set by spontaneous symmetry breaking as in the case of the crystal, the antiferromagnet and the local pairing model for superconductivity [1, 2, 3].

If we substitute values for the temperature and number of Cooper pairs as they are used in modern day Cooper-pair box qubits [4, 5], then the resulting decoherence time τ will be of the order of 500 microseconds. Other sources of decoherence typically destroy superposition states in these qubits long before the limit set by the thin spectrum states is reached. However, with the current rapid advance of technology this new source of decoherence may become important within the near future.

CONCLUSIONS.

In this paper we have shown that the superconducting state of the BCS model is characterized by a broken global U(1) phase symmetry. We have described the process of spontaneous symmetry breaking and the associated thin spectrum in the BCS model for superconductivity. The presence of the thin spectrum states associated with spontaneous symmetry breaking in qubits based on superconducting material, will lead to dephasing and decoherence of these qubits within the time $\tau = 2\pi N\hbar/k_B T$, where N counts the number of Cooper pairs involved in the superposition. This timescale is universal in the sense that it does not depend in the underlying model parameters. Its form coincides precisely with that of the decoherence time induced by thin spectrum dynamics in antiferromagnets and quantum crystals.

The maximum coherence time that we found here for superconducting devices should apply directly to experimental realizations of the so called Cooper pair box qubits, and thus give a maximum coherence time of the order of half a millisecond. The decoherence caused by the thin spectrum at the moment is much weaker than that caused by other sources, but it may well come within experimental reach within the near future.

REFERENCES

1. J. van Wezel, J. van den Brink, and J. Zaanen, *Phys. Rev. Lett.* **94**, 230401 (2005).
2. J. van Wezel, J. Zaanen, and J. van den Brink, *Phys. Rev. B* **74**, 094430 (2006).
3. J. van Wezel, and J. van den Brink, *To Be Published* (2007).
4. Y. Nakamura, Y. A. Pashkin, and J. S. Tsai, *Nature* **398**, 786 (1999).
5. D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve, and M. H. Devoret, *Science* **296**, 886 (2002).
6. P. Nozières, and S. Schmitt-Rink, *J. Low Temp. Phys.* **59**, 195 (1985).
7. P. Anderson, *Phys. Rev.* **112**, 1900 (1958).
8. M. Tinkham, *Introduction to Superconductivity*, McGraw-Hill inc., New York, 1996.
9. J. van Wezel, J. Zaanen, and J. van den Brink, *ArXiv: Physics* p. 0609177 (2006), submitted to AM. J. Phys.