



# Ab Initio determination of Cu 3d orbital energies in layered copper oxides

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It has long been argued that the minimal model to describe the low-energy physics of the high  $T_c$  superconducting cuprates must include copper states of other symmetries besides the canonical  $3d_{x^2-y^2}$  one, in particular the  $3d_{z^2}$  orbital. Experimental and theoretical estimates of the energy splitting of these states vary widely. With a novel *ab initio* quantum chemical computational scheme we determine these energies for a range of copper-oxides and -oxychlorides, determine trends with the apical Cu–ligand distances and find excellent agreement with recent Resonant Inelastic X-ray Scattering measurements, available for  $\text{La}_2\text{CuO}_4$ ,  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ , and  $\text{CaCuO}_2$ .

It is generally accepted that the low-energy physics of the layered Cu oxide compounds in their normal state is reasonably well described by models which incorporate the “in-plane” Cu  $3d_{x^2-y^2}$  and O  $2p_x/2p_y$  orbitals. However, the energy window over which such models provide a qualitatively correct picture is a matter of active research. One additional ingredient which is often invoked is the Cu  $3d_{z^2}$  orbital, perpendicular onto the  $\text{CuO}_2$  layers, and the apical O  $2p_z$  functions having  $\sigma$ -type overlap with the Cu  $3d_{z^2}$ . Recent multi-orbital calculations using dynamical mean-field theory<sup>1</sup> show indeed that some of the features of the optical, x-ray absorption, and photoemission spectra can be better reproduced when the Cu  $3d_{z^2}$  orbitals are explicitly included in the many-body treatment. At finite doping, the inclusion of the Cu  $3d_{z^2}$  functions makes a difference even for the low-energy states close to the Fermi level<sup>1</sup>.

The off-diagonal coupling between states of  $x^2-y^2$  and  $z^2$  symmetry was actually found to substantially affect the dispersion of the low-energy bands and the shape of the Fermi surface in earlier semiphenomenological models<sup>2,3</sup>, density-functional calculations<sup>4</sup>, and quantum chemical studies<sup>5</sup>. Moreover, Ohta *et al.*<sup>6</sup> and recently Sakakibara *et al.*<sup>7</sup> suggested that a direct relation exists between the magnitude of  $T_c$  and the size of the  $d_{x^2-y^2} - d_{z^2}$  splitting. The splittings within the Cu  $3d$  shell are also relevant to excitonic models for pairing and high- $T_c$  superconductivity<sup>8,9</sup>. Even if the importance of the Cu  $3d_{z^2}$  state is stressed in this considerable body of work, the actual experimental and theoretical estimates of the energy of this state vary widely.

Sharp features at about 0.4 eV in early optical measurements on  $\text{La}_2\text{CuO}_4$  and  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$  were initially assigned to crystal-field Cu  $d_{x^2-y^2}$  to  $d_{z^2}$  charge excitations<sup>10</sup>. A different interpretation in terms of magnetic excitations was proposed by Lorenzana and Sawatzky<sup>11</sup> and latter on confirmed by analysis of the resonant inelastic x-ray scattering (RIXS) spectra at the Cu  $K$  and  $L_3$ -edge<sup>12,13</sup>. The RIXS experiments also show that in  $\text{La}_2\text{CuO}_4$  and  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$  the Cu  $d_{x^2-y^2}$  to  $d_{z^2}$  transitions occur at 1.5–2.0 eV<sup>14</sup>, which is substantially larger than the outcome of earlier wavefunction-based quantum chemical calculations, 1.0–1.2 eV<sup>15</sup>, or density-functional estimates, 0.9 eV<sup>7</sup>.

With the aim to settle this point we employ a recently developed *ab initio* quantum chemical computational scheme to extract the splittings within the Cu  $3d$  shell in several layered copper oxides. Excellent agreement is found for  $\text{La}_2\text{CuO}_4$ ,  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ , and  $\text{CaCuO}_2$  with recent RIXS measurements<sup>14</sup>. Further, the  $d_{x^2-y^2}$  to  $d_{z^2}$  excitation energies computed here for  $\text{La}_2\text{CuO}_4$ ,  $\text{YBa}_2\text{Cu}_3\text{O}_6$ , and  $\text{HgBa}_2\text{CuO}_4$  are relevant to models which attempt to establish a direct relation between the relative energy of the out-of-plane  $d_{z^2}$  level and the critical temperature  $T_c$ <sup>7</sup>. In particular, the large difference between the critical superconducting temperatures of doped  $\text{La}_2\text{CuO}_4$  and  $\text{HgBa}_2\text{CuO}_4$  was directly attributed to a large difference between the  $d_{x^2-y^2}$  to  $d_{z^2}$  excitation energies<sup>7</sup>.

## Results

To study bound, excitoniclike states such as the  $d$ - $d$  charge excitations in copper oxides, we rely on real-space *ab initio* methods. In the spirit of modern multi-scale electronic-structure approaches, we describe a given region



around a central Cu site by advanced quantum chemical many-body techniques while the remaining part of the solid is modeled at the Hartree-Fock level. The complete-active-space self-consistent-field (CASSCF) method was used to generate multireference wavefunctions for further configuration-interaction (CI) calculations<sup>16</sup>. In the CASSCF scheme, a full CI is carried out within a limited set of “active” orbitals, i.e., all possible occupations are allowed for those active orbitals. The active orbital set includes in our study all  $3d$  functions at the central Cu site and the  $3d_{x^2-y^2}$  functions of the Cu nearest neighbor (NN) ions. Strong correlations among the  $3d$  electrons are thus accurately described. The final CI calculations incorporate all single and double excitations from the Cu  $3s, 3p, 3d$  and O  $2p$  orbitals on a given  $\text{CuO}_4$  plaquette and from the  $3d_{x^2-y^2}$  orbitals of the Cu NN's. Such a CI treatment is referred to as SDCI. The CASSCF and SDCI investigations were performed with the MOLPRO quantum chemical software<sup>17</sup>.

Both SDCI and RIXS results for the Cu  $d$ -level splittings are listed in Table 1. The relative energies of the peaks observed between 1 and 3 eV in the Cu  $L_3$ -edge RIXS spectra<sup>14</sup> are the sum of a crystal-field contribution, i.e., an on-site crystal-field splitting  $E_{\text{cf}}$ , and a magnetic term  $\Delta E_{\text{mgn}}$ . The quantum chemical calculations have been performed to extract  $E_{\text{cf}}$  for a ferromagnetic (FM) arrangement of the Cu  $d$  spins. A SDCI treatment for an antiferromagnetic (AF) alignment of the Cu spins in the embedded cluster of five Cu sites is computationally not feasible (see Methods for details).  $\Delta E_{\text{mgn}}$  accounts for AF order in the ground-state configuration of the Heisenberg antiferromagnet and is determined as follows. First the value for the NN exchange coupling constant  $J$  is computed by considering an embedded cluster consisting of two  $\text{CuO}_4$  plaquettes. For  $\text{CaCuO}_2$ , for example, we find  $J = 0.13$  eV, in good agreement with the theoretical results reported in Ref. [18] and with values from experimental data<sup>11, 13, 14</sup>. With this value of  $J$  in hand we return to the cluster with five Cu sites and flip the spin of the central Cu ion. This corresponds to an energy increase  $\Delta E = zJ/2 = 2J$ , where  $z = 4$  is the number of NN's and we neglect the quantum fluctuations. For the crystal-field excited states, the superexchange with the NN Cu  $d_{x^2-y^2}$  spins is much weaker for a hole excited into the  $d_{z^2}$  orbital and zero by symmetry for a hole into a  $t_{2g}$  orbital. This contribution due to intersite  $d_{z^2} - d_{x^2-y^2}$  superexchange,  $\Delta E' = 2J'$ , is not included either in the quantum chemical calculations but in a first approximation we can neglect the weak intersite AF interaction  $J'$  involving a  $d_{z^2}$  hole. From overlap considerations,  $J'$  is only a small fraction of the ground-state superexchange  $J$ . For a meaningful comparison between the SDCI and RIXS data, we subtracted in Table 1 from the relative RIXS energies reported in Ref. [14] the term  $\Delta E_{\text{mgn}} = \Delta E - \Delta E' \approx 2J$  representing the magnetic stabilization of the ground-state configuration with respect to the crystal-field excited states. Since  $J \approx 0.13$  eV,  $\Delta E_{\text{mgn}} \approx 0.26$ .

The agreement between our SDCI excitation energies and the results from RIXS is remarkable. As shown in Table 1, the differences between the SDCI and RIXS energies are not larger than 0.15 eV. The

only exception is the splitting between the  $x^2-y^2$  and  $xz/yz$  levels in  $\text{CaCuO}_2$ , where the SDCI value is 0.3 eV larger than in the RIXS measurements. That an accurate description of neighbors beyond the first ligand coordination shell is crucial is clear from the comparison between our and earlier quantum chemical data. In the calculations described in Ref. [15], only one  $\text{CuO}_6$  octahedron or one  $\text{CuO}_5$  pyramid was treated at the all-electron level. Farther neighbors were described by either atomic model potentials or point charges. Deviations of 0.4 and 0.6 eV (up to 50%) for the  $z^2$  levels in  $\text{La}_2\text{CuO}_4$  and  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ <sup>15</sup>, for example, are mainly due to such approximations in the modeling of the nearby surroundings. The  $d$ -level splittings depend after all on the charge distribution at the NN ligand sites. The latter is obviously sensitive to the manner in which other species in the immediate neighborhood are modeled. The quality of the results reported here is directly related to the size of the clusters, i.e., five  $\text{CuO}_4$  plaquettes, all apical ligands plus the NN closed-shell metal ions of the central polyhedron.

Superconductivity has not been observed in  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$  and  $\text{CaCuO}_2$ . The  $d$ -level splittings for three representative cuprate superconductors, i.e.,  $\text{La}_2\text{CuO}_4$ ,  $\text{HgBa}_2\text{CuO}_4$ , and  $\text{YBa}_2\text{Cu}_3\text{O}_6$ , are listed in Table 2. The maximum  $T_c$ 's achieved by doping in these three materials are 35, 95, and 50 K, respectively. For the  $\text{YBa}_2\text{Cu}_3\text{O}_6$  compound, we here refer to the maximum  $T_c$  which can be achieved by Ca doping<sup>19</sup>. The large difference between the critical temperatures in  $\text{La}_2\text{CuO}_4$  and  $\text{HgBa}_2\text{CuO}_4$  was assigned in Ref. 7 to a large difference between the relative energies of the  $z^2$  states in the two materials. The density-functional results for the splittings between the  $x^2-y^2$  and  $z^2$  levels in  $\text{La}_2\text{CuO}_4$  and  $\text{HgBa}_2\text{CuO}_4$  are 0.91 and 2.19 eV, respectively<sup>7</sup>. RIXS data are not available for  $\text{HgBa}_2\text{CuO}_4$  and independent estimates for the energy separation between the  $x^2-y^2$  and  $z^2$  states are therefore desirable. While we find a rather similar value for  $\text{HgBa}_2\text{CuO}_4$ , of 2.09 eV, the quantum chemical and RIXS results<sup>14</sup> for  $\text{La}_2\text{CuO}_4$  are substantially larger, about 1.4 eV. This makes the difference between the  $d$ -level splittings in the above mentioned compounds less spectacular, i.e.,  $E_{z^2}^{\text{HBCO}} - E_{z^2}^{\text{LCO}}$  is reduced from 1.3 eV in Ref. [7] to 0.7 eV in the present study, which suggests that the model constructed and the conclusions drawn in Ref. [7] at least require extra analysis.

The distance between the Cu and apical ligand sites increases from 2.40 Å in  $\text{La}_2\text{CuO}_4$ <sup>20</sup> to 2.78 Å in  $\text{HgBa}_2\text{CuO}_4$ <sup>21</sup>. The effect of this growth of the apical Cu–O bond length on the relative energy of the  $z^2$  hole state can be understood by using simple electrostatic arguments: when the negative apical ions are closer to the Cu site, less energy is needed to promote the Cu  $3d$  hole into the  $z^2$  orbital pointing toward those apical ligands. For  $\text{HgBa}_2\text{CuO}_4$ , the lowest crystal-field excitation is therefore to the  $xy$  level and requires about 1.3 eV, see Table 2, while the  $z^2$  and  $xz/yz$  levels are nearly degenerate and more than 0.5 eV higher in energy. On the other hand, in  $\text{La}_2\text{CuO}_4$  the lowest crystal-field excitation is to the  $z^2$  orbital, see Table 1. Our results also reproduce the near degeneracy between the  $z^2$  and  $xy$  levels in  $\text{La}_2\text{CuO}_4$ , as found in the RIXS experiments. In  $\text{CaCuO}_2$ , there are no apical ligands. The splitting between the  $x^2-y^2$  and  $z^2$  levels is therefore the largest for  $\text{CaCuO}_2$ , about 2.4 eV, see Table 1.

**Table 1 | CASSCF+SDCI versus RIXS results for the Cu  $d$ -level splittings in  $\text{La}_2\text{CuO}_4$ ,  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ , and  $\text{CaCuO}_2$  (eV). The ground-state Cu  $t_{2g}^6 d_{z^2}^1 d_{x^2-y^2}^1$  configuration is taken as reference. A  $2J$  term was here subtracted from each of the RIXS values reported in Ref. [14], see text.**

Hole orbital	$\text{La}_2\text{CuO}_4$ SDCI/RIXS	$\text{Sr}_2\text{CuO}_2\text{Cl}_2$ SDCI/RIXS	$\text{CaCuO}_2$ SDCI/RIXS
$x^2-y^2$	0	0	0
$z^2$	1.37/1.44	1.75/1.71	2.38/2.39
$xy$	1.43/1.54	1.16/1.24	1.36/1.38
$xz, yz$	1.78/1.86	1.69/1.58	2.02/1.69

**Table 2 | Cu  $d$ -level energy splittings for  $\text{La}_2\text{CuO}_4$ ,  $\text{HgBa}_2\text{CuO}_4$ , and  $\text{YBa}_2\text{Cu}_3\text{O}_6$  (eV). CASSCF+SDCI calculations for 5-plaquette FM clusters.**

Hole orbital	$\text{La}_2\text{CuO}_4$	$\text{HgBa}_2\text{CuO}_4$	$\text{YBa}_2\text{Cu}_3\text{O}_6$
$x^2-y^2$	0	0	0
$z^2$	1.37	2.09	1.47
$xy$	1.43	1.32	1.22
$xz, yz$	1.78	1.89	1.57

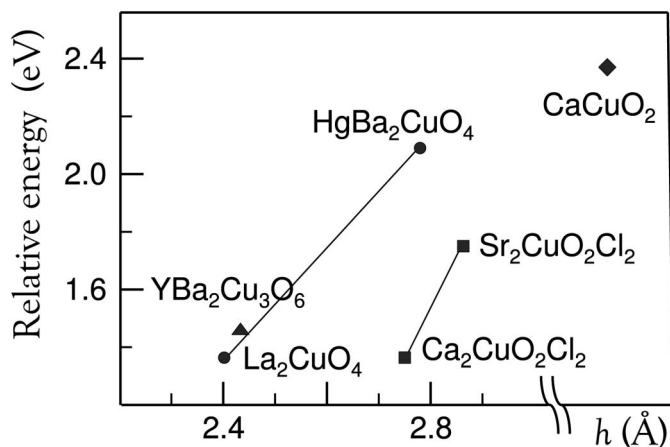


## Discussion

The parameter that plays the major role in determining the size of the  $d$ -level splittings in layered cuprates is clearly the apical Cu–ligand distance. There are, however, few other factors which come into play such as the number and nature of the apical ligands, the in-plane Cu–O bond lengths, buckling of the  $\text{CuO}_2$  planes, and the configuration of the farther surroundings. Trends concerning the relative energy of the  $z^2$  hole state in different cuprates are illustrated in Fig. 1, which includes data for systems having one apical O site ( $\text{YBa}_2\text{Cu}_3\text{O}_6$ ), two apical O's ( $\text{La}_2\text{CuO}_4$ ,  $\text{HgBa}_2\text{CuO}_4$ ), two apical Cl ions ( $\text{Ca}_2\text{CuO}_2\text{Cl}_2$ ,  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ ) or no apical ligand ( $\text{CaCuO}_2$ ). The apical Cu–O distances in  $\text{La}_2\text{CuO}_4$  and  $\text{YBa}_2\text{Cu}_3\text{O}_6$ , for example, are nearly the same, 2.40 vs. 2.45 Å<sup>19, 20, 22</sup>. In  $\text{YBa}_2\text{Cu}_3\text{O}_6$ , however, there is a single apical O. For this reason the  $z$  hole state is somewhat destabilized in  $\text{YBa}_2\text{Cu}_3\text{O}_6$  and lies above the  $xy$  hole configuration, see Table 2. Yet since the Cu ion is shifted towards the apical ion, out of the basal O plane, the  $x^2-y^2$  hole state is also destabilized such that the splitting between the  $x^2-y^2$  and  $z^2$  levels is finally close to the value found in  $\text{La}_2\text{CuO}_4$ . Further, the apical Cu–ligand distances are slightly larger in  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$  as compared to  $\text{HgBa}_2\text{CuO}_4$ , 2.86 vs. 2.78 Å, respectively. The apical ions also have a smaller effective charge in  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ , which should lead to a larger relative energy of the  $z^2$  hole state in  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$  as compared to  $\text{HgBa}_2\text{CuO}_4$ . The fact that the relative energy of the  $z^2$  hole state is actually larger in  $\text{HgBa}_2\text{CuO}_4$ , see Fig. 1, must be related to the smaller in-plane Cu–O distances in  $\text{HgBa}_2\text{CuO}_4$ , 1.94 in  $\text{HgBa}_2\text{CuO}_4$  vs. 1.99 Å in  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ , which stabilizes the ground-state  $x^2-y^2$  hole configuration in the former compound, and to farther structural details. From  $\text{Ca}_2\text{CuO}_2\text{Cl}_2$  to  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ , the Cu–Cl separation increases from 2.75 to 2.86 Å<sup>23, 24</sup> and the energy of the  $z^2$  level from 1.37 to 1.75 eV.

In contrast to the  $z^2$  orbitals, the relative energies of the  $xy$  levels display much smaller variations, in an interval of 1.2–1.5 eV, see Tables 1 and 2. Substantially smaller are also the variations computed for the  $xz/yz$  levels, in an energy window between 1.6 and 2.0 eV.

To summarize, we employ state of the art quantum chemical methods to investigate the Cu  $3d$  electronic structure of layered Cu oxides. Multiconfiguration and multireference configuration-interaction calculations are carried out on finite clusters including five  $\text{CuO}_4$  plaquettes plus additional apical ligand and closed-shell metal ion NN's. The localized Wannier functions attached to these atomic sites are obtained from prior Hartree-Fock computations for the periodic system. Excellent agreement is found between our theoretical results and recent Cu  $L_3$ -edge RIXS data for  $\text{La}_2\text{CuO}_4$ ,



**Figure 1** | Relative energy of the Cu  $z^2$  level as function of the distance  $h$  between the Cu and apical ligands in different cuprates. There are two apical O sites in  $\text{La}_2\text{CuO}_4$  and  $\text{HgBa}_2\text{CuO}_4$ , two apical Cl sites in  $\text{Ca}_2\text{CuO}_2\text{Cl}_2$  and  $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ , and one apical O ion in  $\text{YBa}_2\text{Cu}_3\text{O}_6$ . In  $\text{CaCuO}_2$  the apical ligands are absent.

$\text{Sr}_2\text{CuO}_2\text{Cl}_2$ , and  $\text{CaCuO}_2$ . RIXS is a novel experimental tool to investigate both magnetic and charge excitations with high resolution and accuracy. Our computational scheme and present results indicate a promising route for the modeling and reliable interpretation of RIXS spectra in correlated  $3d$ -metal compounds. A next step along this path is the computation of transition probabilities and intensities at the *ab initio* level, which requires the explicit calculation of the intermediate Cu  $3p$  core hole wavefunctions.

Further, the excitation energies computed here for  $\text{La}_2\text{CuO}_4$ ,  $\text{YBa}_2\text{Cu}_3\text{O}_6$ , and  $\text{HgBa}_2\text{CuO}_4$  are relevant to models which attempt to establish a direct relation between the critical temperature  $T_c$  and the strength of the  $(x^2-y^2)-z^2$  coupling. For  $\text{La}_2\text{CuO}_4$ , in particular, the density-functional estimate used as input parameter in such models<sup>7</sup> is about 0.5 eV smaller than our result. Consequently, the difference we find between the  $d_{x^2-y^2} - d_{z^2}$  splittings in  $\text{La}_2\text{CuO}_4$  and  $\text{HgBa}_2\text{CuO}_4$  is less spectacular as compared to the value reported by Sakakibara *et al.*<sup>7</sup>, suggesting a reevaluation of the analysis in Ref. [7].

## Methods

The first step in our study is a restricted Hartree-Fock (RHF) calculation for the ground-state configuration of the periodic system. The RHF calculations are performed with the CRYSTAL package<sup>25</sup>. We employed experimental lattice parameters<sup>14, 20–24</sup> and Gaussian-type atomic basis sets, i.e., triple-zeta basis sets from the CRYSTAL library for Cu, O, and Cl plus basis sets of either double-zeta or triple-zeta quality for the other species. Post Hartree-Fock many-body calculations are subsequently carried out on finite clusters, which are sufficient because of the local character of the correlation hole. They consist of five  $\text{CuO}_4$  plaquettes, i.e., a “central”  $\text{CuO}_4$  unit plus the four NN plaquettes. When present, the apical ligands, oxygen or chlorine, are incorporated as well in the finite cluster  $C$ . Additionally, the finite cluster  $C$  includes in each case the NN closed-shell metal ions around the “central” Cu site. In  $\text{La}_2\text{CuO}_4$ , for example, there are ten  $\text{La}^{3+}$  NN's. In  $\text{YBa}_2\text{Cu}_3\text{O}_6$ , there are one  $\text{Cu}^{1+}$   $3d^9$ , four  $\text{Y}^{3+}$ , and four  $\text{Ba}^{2+}$  NN's.

The orbital basis entering the post Hartree-Fock correlation treatment is a set of projected RHF Wannier functions: localized Wannier orbitals (WO's) are first obtained with the Wannier-Boys localization module<sup>26</sup> of the CRYSTAL package and subsequently projected onto the set of Gaussian basis functions associated with the atomic sites of  $C$ <sup>27</sup>. Moreover, the RHF data is used to generate an effective embedding potential for the five-plaquette fragment  $C$ . This potential is obtained from the Fock operator in the RHF calculation<sup>27</sup> and models the surroundings of the finite cluster, i.e., the remaining of the crystalline lattice.

The central  $\text{CuO}_4$  plaquette and the four NN Cu sites form the active region of the cluster, which we denote as  $C_A$ . The other ions in  $C$ , i.e., each ligand coordination cage around the four Cu NN's and the NN closed-shell metal ions, form a buffer region  $C_B$  whose role is to ensure an accurate description of the tails of the WO's centered in the active part  $C_A$ <sup>27</sup>. For our choice of  $C_B$ , the norms of the projected WO's centered within the active region  $C_A$  are not lower than 99.5% of the original crystal WO's. While the occupied WO's in the buffer zone are kept frozen, all valence orbitals centered at O and Cu sites in  $C_A$  (and their tails in  $C_B$ ) are further reoptimized in multiconfiguration CASSCF calculations. In the latter, the ground-state wavefunction and the lowest four crystal-field excited states at the central Cu site are computed simultaneously in a state-averaged multiroot calculation<sup>16</sup>. The  $d$ -level splittings at the central Cu site are finally obtained at the CASSCF+SDCI level of theory as the relative energies of the crystal-field excited states. The virtual orbital space in the multireference SDCI calculation cannot be presently restricted just to the  $C_A$  region. It thus includes virtual orbitals in both  $C_A$  and  $C_B$ , which leads to very large SDCI expansions,  $\sim 10^9$  Slater determinants for a FM configuration. For this reason, we restrict the CASSCF+SDCI calculations to FM alignment of the Cu  $d$  spins.

The effective embedding potential is added to the one-electron Hamiltonian with the help of the CRYSTAL-MOLPRO interface program<sup>28</sup>. Although the WO's at the atomic sites of  $C$  are derived for each of the compounds discussed here by periodic RHF calculations for the Cu  $3d^9$  electron configuration, the embedding potentials are obtained by replacing the  $\text{Cu}^{2+} 3d^9$  ions by closed-shell  $\text{Zn}^{2+} 3d^{10}$  species. This is a good approximation for the farther  $3d$ -metal sites, as the comparison between our results and RIXS data shows. An extension of our embedding scheme toward the construction of open-shell embeddings is planned for the near future.

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## Author contributions

L.H., P.F., and J.v.d.B. wrote the main manuscript text, L.H. and L.S. prepared figure 1. All authors reviewed the manuscript.

## Additional information

**Competing Financial Interests:** Authors have no Competing Financial Interests.

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