Josephson Currents Induced by the Witten Effect

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We reveal the existence of a new type of topological Josephson effect involving type II superconductors and three-dimensional topological insulators as tunnel junctions. We predict that vortex lines induce a variant of the Witten effect that is the consequence of the axion electromagnetic response of the topological insulator: at the interface of the junction each flux quantum attains a fractional electrical charge of $e/4$. As a consequence, if an external magnetic field is applied perpendicular to the junction, the Witten effect induces an ac Josephson effect in the absence of any external voltage. We derive a number of further experimental consequences and propose potential setups where these quantized, flux induced Witten effects may be observed.

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One distinguished feature of topological superconductivity is that it hosts gapless boundary states in the form of Majorana fermions, i.e., particle states that are their own antiparticles [1]. One way of indirectly detecting Majorana fermions, and therefore the topological nature of superconductivity in the system, was proposed by Kitaev a long time ago in the framework of a simple, exactly solvable model [2]. Kitaev pointed out that a semiconducting wire with strong spin-orbit coupling, where p-wave-like superconductivity is induced by the proximity effect to an s-wave superconductor (SC), could host unpaired Majorana modes at its ends, provided the chemical potential does not exceed the energy gap between the elementary excitations. Within this framework, a tunnel junction between two Kitaev wires would feature fused Majorana modes and lead to a fractional (4π-periodic) Josephson effect. Since Kitaev's seminal paper, several papers have discussed further effects and the possible use of Majorana fermions as the means to realizing topologically protected quantum computation [3]. Experimental evidence for a fractional ac Josephson effect has been reported in a hybrid InSb/Nb nanowire [4], thus providing evidence for fused Majorana states.

Another possibility to realize a fractional Josephson effect is having a topological insulator (TI) as a tunnel junction [5]. In this case, a fractional Josephson effect due to fused Majorana fermions also emerges as a consequence of the proximity effect. Recently, the ac Josephson effect has also been measured in this case using HgTe as the three-dimensional insulator junction [6]. Despite the recent progress in the measurement of Josephson effect phenomena related to Majorana physics, the true topological character has yet to be clearly demonstrated. For instance, it remains to be shown that non-Abelian statistics can be realized in some way by means of Josephson junctions featuring fused Majorana modes, which would pave the way to implement quantum information processing [7].

In this Letter, we show that another type of topological Josephson effect is also present in SC-TI-SC junctions, when an external magnetic field is applied perpendicular to the junction and the superconductor is a type II one. We will show that the induced vortex lines act as magnetic monopoles in the sense that they trigger a variant of the Witten effect, which in a field theory setting endows magnetic monopoles with a fractional electric charge [8]. In our condensed matter setting, the resulting electrical charge of magnetic vortices is the consequence of the axion electromagnetic response of the topological insulator [9]. This has a number of experimentally accessible consequences. As the vortex lines perpendicular to the TI junction become electrically polarized, they may trigger an ac Josephson effect in the absence of an external voltage. Furthermore, the Josephson frequency will turn out to be quantized, as a consequence of a Berry phase for the tunnel junction induced by the Witten effect. This will, in turn, imply a peculiar behavior for Shapiro steps. These Josephson-Witten effects are expected to be rather robust as they merely rely on magnetic fluxes traversing the SC-TI boundary and do not require any further fine-tuning or interplay between different order parameters.

Electromagnetic variant of the Witten effect.—For a strong three-dimensional TI, it was shown [9] that the electromagnetic response leads to a so-called axion term [10] in the effective electromagnetic Lagrangian

$$\mathcal{L}_{\rm axion} = \frac{e^2\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B}, \tag{1}$$

where units are such that $\hbar = c = 1$. Generally, θ is a field, the so-called axion. Several properties are important in the following [11]: (i) The value of θ is defined modulo 2π;

(ii) within the bulk, θ is constant; (iii) for a uniform θ, the above Lagrangian is a total derivative, yielding henceforth a surface term in the action; and (iv) in the presence of time-reversal symmetry (TRS), θ can either be 0 (the topologically trivial case) or π, in particular, for a TRS strong topological insulator $\theta = \pi$. However, in the absence of TRS, θ need not be quantized, and a TI can adiabatically be transformed into a band insulator. Since the axion term is a surface term, it does not changes the field equations. However, it does change the boundary conditions. This point is crucial and leads to the Witten effect [8]: when magnetic monopoles are present, the total electric charge becomes fractionalized due to the magnetic monopoles acquiring an electrical charge.

The Witten effect has been originally predicted for an $O(3)$ Higgs model, which has magnetic monopole solutions in the spontaneous symmetry breaking regime as demonstrated by 't Hooft and Polyakov [12]. However, in electrodynamics, magnetic monopoles have to be added by hand. Nevertheless, in the presence of the axion term, the Witten effect also holds [10] just as in the more general case originally discussed by Witten. Furthermore, as we will show in the following, a form of the Witten effect also holds for the case of vortex lines. To this end, let us consider the simplest action for electrodynamics with an axion term

$$S = \int dt \int d^3x \left(\frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) + \frac{e^2\theta}{4\pi^2}\mathbf{E}\cdot\mathbf{B} - \rho\phi - \mathbf{j}\cdot\mathbf{A} \right). \tag{2}$$

In a system without boundaries, the Witten effect readily follows from the integral form of the Gauss law in the presence of the θ term

$$4\pi Q = \oint_S d\mathbf{S}\cdot\mathbf{E} = 4\pi\left(q - \frac{e^2\theta}{4\pi^2}\int_V d^3r \nabla\cdot\mathbf{B} \right), \tag{3}$$

where q is the electrical charge, Q is the total charge, and the volume integral is bounded by the surface S. In standard electrodynamics, $\nabla\cdot\mathbf{B} = 0$, such that for constant θ nothing happens, leading to $Q = q$. If we assume that the theory has magnetic monopoles, the standard Witten effect [8] arises. Using that the electrical charge $q = ne (n \in \mathbb{Z})$ and the flux of a single monopole $\Phi_B = 2\pi/e$, the total charge is

$$Q = e\left(n - \frac{\theta}{2\pi} \right). \tag{4}$$

In the electrodynamics of condensed matter systems, magnetic monopoles do not arise. However, the presence of a TI surface prompts θ to change, and the Gauss law in the form $\nabla\cdot\mathbf{E} = 4\pi\rho + (e^2/\pi)\nabla\theta\cdot\mathbf{B}$ needs to be used in order to accommodate this change. Let us now specifically consider the situation of a half-space ($z < 0$) occupied with a type II superconductor interfacing with a strong TI on the upper half-space ($z > 0$); see Fig. 1(a). An external field $\mathbf{H}_{\rm ext}$ perpendicular to the interface $z = 0$ generates magnetic vortices in the superconductor. Inside the TI and near the interface, there are stray fields originating from the vortex lines with the boundary condition that $\mathbf{B} = \mathbf{H}_{\rm ext}$ for $z \to \infty$. We have that $\theta(z) = 0$ for $z < 0$ (i.e., inside the superconductor), while θ is uniform in the TI, including its surface at $z = 0$. For straight vortex lines, the magnetic field inside the superconductor depends only on the in-plane radial coordinate, and we find for the charge at the $z = 0$ interface

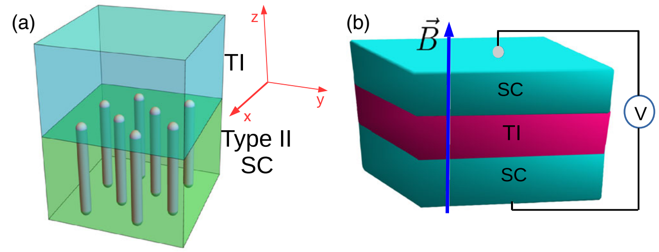

FIG. 1. (a) Schematic view of a junction between a strong TI (e.g., Bi_2Se_3, Bi_2Te_3, or strained HgTe) and a type II SC (e.g., Nb,V, or a high T_c cuprate). The magnetic flux from the vortex lines at the interface induces a charge fractionalization due to the Witten effect. (b) Schematics of a possible experimental setup to measure the Josephson-Witten effect. The Witten effect acts on the vortex lines rather than on magnetic monopoles, thus creating a potential difference across the junctions.

$$\begin{aligned} Q &= q + \frac{e^2}{4\pi^2}\int d^2r B(r) \int_{-\infty}^0 dz \frac{d\theta}{dz} \\ &= q + \frac{e^2\theta}{4\pi^2}\Phi_B, \end{aligned} \tag{5}$$

where now $\Phi_B = N_v\Phi_0 = N_v\pi/e$ is the total flux due to N_v flux lines. Here, $\Phi_0 = 2\pi/e^* = \pi/e$ is the elementary flux quantum associated with the Cooper pair ($e^* = 2e$). With d the thickness of the TI, a charge $q - (e^2\theta/4\pi^2)\Phi_B$ is similarly found at the (top) $z = d$ SC-TI interface. This analysis implies that when a TI shares an interface with a type II superconductor, the flux of the vortex lines becomes electrically polarized by the Witten effect.

At the interface of a SC and a strong TI with time-reversal symmetry $\theta = \pi$, the Witten effect endows each magnetic flux with a charge $e/4$. Since a vortex is a solitonic object, it is allowed to carry a fractional electrical charge, similar to the situation found with magnetic monopoles (also solitonic objects) in the standard Witten effect. It is interesting that the statistical properties of these fractionally charged fluxes might, in principle, be determined by shot noise [13] or interferometry [14] experiments.

Microscopically, the origin of this charge fractionalization can be traced back in part to the origin of the axion term in the Lagrangian of a three-dimensional TI [9]. In this case, a well-known argument shows that the electromagnetic response implies a half-quantized Hall conductivity σ_{xy} for the surface electrons [9]. Thus, we can adapt a simple argument by McGreevy [15] estimating the amount of charge ΔQ acquired by a localized flux to our case as follows. Applying the Faraday law to the elementary flux quantum $\Phi_0 = \pi/e$ of a vortex and noting that there are circulating Hall currents at the vortex core, we obtain

$$\Phi_0 = \frac{\pi}{e} = -\int dt \oint d\mathbf{r}\cdot\mathbf{E} = -\frac{\Delta Q}{\sigma_{xy}}, \tag{6}$$

where we have used that the transverse current in the vortex core $j_r = \sigma_{xy}E_\varphi$. Since $\sigma_{xy} = -e^2/(4\pi)$ [this is the value of the half-quantized Hall conductivity $\sigma_{xy} = e^2/(2h)$ when $\hbar = 1$], the above simple argument yields $\Delta Q = e/4$.

The discussion above can be further elaborated to enable us to consider time-dependent magnetic fluxes, yielding another instance of the Witten effect. In this case, the action (2) becomes

$$S = \int dt \int d^3x \left[\frac{1}{8\pi}(\mathbf{E}^2 - \mathbf{B}^2) - \rho\phi - \mathbf{J}\cdot\mathbf{A} - \frac{e^2}{4\pi^2}\nabla\cdot(\theta\mathbf{E}\times\mathbf{A})\right], \tag{7}$$

where the total current density

$$\mathbf{J} = \mathbf{j} - \frac{e^2}{4\pi^2}(\nabla\theta\times\mathbf{E}) - \frac{e^2\theta}{4\pi^2}\nabla\times\mathbf{E}. \tag{8}$$

In the above equation, the term $\sim(\nabla\theta\times\mathbf{E})$ contributes to the Hall conductivity while the term $\sim\theta\nabla\times\mathbf{E}$ contributes to the Witten effect. For example, if $\mathbf{E}$ is uniform and applied in the x direction parallel to the TI surface, we obtain the Hall current along the y direction

$$j_y^H = \frac{e^2E}{4\pi^2}\int_0^d dz\frac{d\theta}{dz} = \frac{e^2E}{4\pi^2}[\theta(d) - \theta(0)], \tag{9}$$

where d is the thickness of the TI. On the other hand, if an electric field is induced due to an external magnetic field applied perpendicular to the surface, the total electric current flowing through one of the surfaces is given by

$$\frac{dQ}{dt} = -\int_S d\mathbf{S}\cdot\mathbf{j} + \frac{e^2\theta}{4\pi^2}\int_S d\mathbf{S}\cdot(\nabla\times\mathbf{E}) = \frac{dq}{dt} + \frac{e^2\theta}{4\pi^2}\oint_C d\mathbf{r}\cdot\mathbf{E} = \frac{dq}{dt} - \frac{e^2\theta}{4\pi^2}\frac{d\Phi_B}{dt}, \tag{10}$$

where we invoked Faraday's law. Equation (10) thus indeed extends our Witten effect results to the time-dependent domain. This analysis implies that when a TI shares an interface with a type II superconductor, the flux of the vortex lines becomes electrically polarized by the Witten effect.

Capacitance due to Witten effect.—The magnetic flux carrying a fractionalized charge due to the Witten effect highlights the interplay between topology and boundary conditions, since the axion term is a total derivative. An immediate consequence of this interplay is the modification of the capacitance energy. One expects that due to the Witten effect, a capacitor can become electrically charged by magnetic fluxes in the plates. Indeed, an application of the Gauss law to a parallel plate capacitor yields the magnitude of the electric field

$$E = \frac{4\pi}{A\epsilon}\left(q - \frac{e^2\theta}{4\pi^2}q_m\right), \tag{11}$$

where ϵ is the dielectric constant of the material filling the capacitor, q_m is the total magnetic monopole charge, and A is the area of the plate. Thus, irrespective of the plate separation, we obtain the voltage difference

$$\Delta V = \frac{1}{C}\left(q - \frac{e^2\theta}{4\pi^2}q_m\right), \tag{12}$$

where C is the usual capacitance of a parallel plate capacitor. We see that for nonzero q_m, the voltage difference is nonzero even if there are no electric charges in the capacitor. One important feature of this capacitor is that the monopole charges in one plate are connected by vortex strings to the opposite monopole charges on the other plate. The Witten effect implies that this vortex string is electrically polarized.

Josephson-Witten effect.—Having established that magnetic monopoles and fluxes can generate an electrical potential difference in the presence of a varying axion field, we consider a SC-TI-SC Josephson junction in this context. Magnetic monopoles are, of course, absent in this physical situation. The Hamiltonian of a Josephson junction [16] features the charging or capacitive energy $C(\Delta V)^2/2$ and the Josephson potential energy $-E_J\cos\Delta\phi$. The voltage difference can be related to the variation of the particle number Δn, which is semiclassically conjugate to the phase difference $\Delta\phi$ via $\Delta V = (2e/C)\Delta n$. Thus [16],

$$H_J = \frac{2e^2}{C}(\Delta n)^2 - E_J\cos\Delta\phi. \tag{13}$$

For a SC-TI-SC Josephson junction, an additional surface energy has to be included due to the axion term, and the Hamiltonian of the junction modifies to

$$H_J = \frac{2e^2}{C}\left(\Delta n + \frac{e\theta}{8\pi^2}\Phi_B\right)^2 - E_J\cos\Delta\phi, \tag{14}$$

since due to the θ term a charging energy is induced when an external magnetic field is applied perpendicular to the junction. As usual, Δn is the Cooper pair number variation that is conjugate to the phase difference $\Delta\phi$ across the junction. Therefore, $i[H_J, \Delta\phi] = \partial_t \Delta\phi = 2e\Delta V_{\rm ind}$ with the induced potential drop

$$\Delta V_{\rm ind} = \frac{1}{C}\left(2e\Delta n + \frac{e^2\theta}{4\pi^2}\Phi_B\right). \tag{15}$$

Since Φ_B is quantized in multiples of $\Phi_0 = \pi/e$, we see from Eqs. (14) and (15) that the original 2π periodicity of θ, which is intrinsic to TI electrodynamics [9], becomes an 8π periodicity, upon proximity with the superconductor. Indeed, we obtain that for arbitrary N_v that both Eqs. (14) and (15) are invariant under $\theta \to \theta + 8\pi$, $\Delta n \to \Delta n - N_v$. This periodicity is also reflected in the excitation spectrum of the Hamiltonian (14) in the case where the charging energy dominates over the Josephson energy E_J. In this case, an elementary second-order perturbation theory calculation yields

$$E_n(\theta) \approx \frac{2e^2}{C}\left(n + \frac{\theta}{8\pi}N_v\right)^2 + \frac{(E_J C)^2/(2e^4)}{4(n + \frac{\theta}{8\pi}N_v)^2 - 1}, \tag{16}$$

where $n \in \mathbb{Z}$. For $\theta = 0$, Eq. (16) just reduces to the perturbation solution of the two-dimensional Stark rotator. For $\theta \neq 0$, the problem is equivalent to one of a particle moving on a ring with a magnetic flux $\Phi = \theta N_v/4$, with a tunnel barrier [17]. This equivalence implies that for $N_v = 1$, an electron going around the ring with a single vortex will effectively only pick up a flux $\theta/4$, which in the case of a time-reversal-invariant system means a $\pi/4$ flux. From this point of view, gauge invariance and the 8π periodicity of θ imply the invariance of the spectrum (16) under the transformation $n \to n + m$, $\theta \to \theta - 8\pi m/N_v$. This result holds exactly, being independent of the perturbative calculation, as it is a consequence of the gauge invariance of the system.

Let us estimate the Witten effect contribution to the voltage drop from Eq. (15). Candidate materials are structures involving either Bi_2Se_3-vanadium or Bi_2Te_3-niobium interfaces. In the absence of external voltages, the voltage drop is due uniquely to the Witten effect induced by the vortices, yielding $\Delta V_{\rm ind} = N_v e/(4C)$. Generally, C represents the combined capacitance between TI and superconductor. For a small junction, the capacitance is in the order of ~1 pF [18]. A magnetic field of ~10 T gives rise to vortices with a typical spacing of 100 nm, which then induce a voltage drop ~2μV, a drop that is comparable to those found in larger junctions and which is easily within the experimental limit of detection.

The ac Josephson effect oscillation frequency exhibits an additional contribution due to the θ term. Thus, the total frequency for the ac Josephson effect is $\omega = \omega_0 + \omega_\theta$, where $\omega_0 = 2eV_0$ is the usual Josephson frequency with $V_0 = 2e\Delta n/C$, and

$$\omega_\theta = \frac{e^3\theta}{2\pi^2 C}\Phi_B \tag{17}$$

is the contribution to the frequency which is induced by the Witten effect. Thus, similarly to the Shapiro steps [19], we find a dc Josephson effect by tuning the voltage such that $\omega_0 = -\omega_\theta$. Using our estimate for the voltage drop at high magnetic fields, we obtain $\omega_\theta \sim 1$ GHz. Thus, in the absence of external voltage, a topological magnetoelectric contribution is still present, implying the ac Josephson current

$$I_J(\Delta\phi, t) = 2eE_J \sin(\Delta\phi + \omega_\theta t). \tag{18}$$

The Witten effect has further important consequences if we consider the Josephson effect in small junctions. To see this, let us consider the partition function of the Lagrangian $L_J = \Delta n \partial_t \Delta\phi - H_J$ in terms of a path integral in imaginary time

$$Z = \int \mathcal{D}\Delta n \mathcal{D}\Delta\phi e^{-\int_0^\beta d\tau (i\Delta n \partial_\tau \Delta\phi + H_J)}, \tag{19}$$

where we have used the fact that Δn is canonically conjugate to $\Delta\phi$. Because of the periodicity of $\Delta\phi$, the above path integral is calculated with a periodic boundary condition taking the form $\Delta\phi(\beta) - \Delta\phi(0) = 2\pi n_W$, and $n_W \in \mathbb{Z}$ is the standard winding number that arises in the partition function of small junctions [20] and which is due to $\Delta\phi$ being conjugate to the particle number operator. By performing the shift $\Delta n \to \Delta n - e\theta\Phi_B/(8\pi^2)$, Eq. (19) acquires a phase factor $e^{ie\theta\Phi_B/(8\pi^2)\int_0^\beta d\tau \partial_\tau \Delta\phi}$, which contains the integral over a total derivative. Because of the boundary condition,

$$\theta = \frac{8\pi m}{N_v}, \qquad m \in \mathbb{Z}, \tag{20}$$

and the 8π periodicity of θ corresponds to the translation $m \to m + N_v$. This result implies that

$$\omega_\theta = \frac{(2e)^2 m}{C}, \tag{21}$$

leading to a quantized dc component. Thus, due to the Witten effect, the voltage is quantized in a way similar to the Shapiro steps [19]. This result modifies, in turn, the way the actual Shapiro steps behave, since now the phenomenon will be characterized by two integers. Indeed, by considering an additional ac voltage $V(t) = V_0 + V_1 \cos(\omega_1 t)$, the standard argument for the Shapiro steps [16] implies the dc voltages

$$V_{nm} = \frac{n\omega_1}{2e} - \frac{2me}{C}, \tag{22}$$

in which case the usual Shapiro step result is obtained for $m = 0$. Equation (22) leads to a charge lattice $Q_{nm} = CV_{nm}$ reminiscent of the Schwinger result [21], generalizing the Dirac quantization to dyons, namely, dipoles involving an electric and a magnetic charge. Similarly to that case, we can express the charge obtained from the voltage (22) in terms of modular transformations [22] describing a so-called S duality [23]. There is also a similarity between Eqs. (22) and (12), when the Dirac duality relation $qq_m = 2\pi$ is accounted for.

A possible experimental setup to test the above prediction on the quantization of the Shapiro steps is shown in Fig. 1(b). The strategy to observe the result (22) is to measure the $I - V$ characteristics using microwave radiation and an external magnetic field perpendicular to the junction. For the case of a resistively shunted junction, $I = I_{c0}\sin\Delta\phi + R^{-1}(V + 2em/n_W) + CdV/dt$, where $I_{c0} = 2eE_J$ is the critical current in absence of the axion term. Thus, the small capacitance regime is shown to obey the differential equation

$$\frac{d\Delta\phi}{dt} = 2eI_{c0}R\left[\frac{1}{I_{c0}}\left(I - \frac{2me}{RC}\right) - \sin\Delta\phi\right], \qquad (23)$$

which yields the time-averaged voltage

$$V_m = R\sqrt{\left(I - \frac{2me}{RC}\right)^2 - I_{c0}^2}, \qquad (24)$$

which implies a quantized critical current. Since the capacitance is small, we obtain for small I and I_{c0} a quantized voltage $V_m \approx 2me/C$.

Conclusions.—We have shown that new types of Josephson effects can arise in tunnel junctions between type II superconductors and strong topological insulators because the axion electromagnetic response of the TI causes a Witten effect that endows magnetic fluxes at the SC-TI interface with an electrical charge. This charge is $e/4$ per elementary flux quantum. As vortex lines perpendicular to the SC-TI junction become electrically polarized, they, in turn, generate an ac Josephson effect in the absence of an external voltage. Furthermore, the Witten effect contributes directly to the Josephson frequency, which, in turn, modifies the way the Shapiro steps behave, a phenomenon that is now characterized by two integers. One expects these Josephson-Witten effects to be rather robust experimentally, as they in the end only rely on magnetic fluxes traversing the SC-TI boundary and do not require any further fine-tuning.

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